Hydroinformatik - SoSe 2026

UW-BHW-414-11: Grundlagen der Kontinuumsmechanik

Prof. Dr.-Ing. habil. Olaf Kolditz

[1]Helmholtz Centre for Environmental Research – UFZ, Leipzig

[2]Technische Universität Dresden – TUD, Dresden

[3]Center for Advanced Water Research – CAWR

[4]TUBAF-UFZ Center for Environmental Geosciences – C-EGS, Freiberg / Leipzig

Dresden, 12.06.2026

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Zeitplan: Hydroinformatik I+II

Sommersemester 2026: Stand: 06.04.2026

Nr.	KW	Datum	ID	Thema
01+02	16	17.04.2026	UW-BHW-414-01/02	Einführung in die Vorlesung, Umweltinformatik
03	16	17.04.2026	UW-BHW-414-03	Werkzeuge, Hello World (in C++)
05	17	24.04.2026	UW-BHW-414-04	Selbststudium: Software-Installationen
07	19	08.05.2026	UW-BHW-414-05	Objekt-Orientierte Programmierung: C++, Klassen
09	20	15.05.2026	UW-BHW-414-06	Programmiersprache Python
11	21	22.05.2026	UW-BHW-414-07/08	Modellierung, Digitalisierung - Wasser 4.0
00	22	29.05.2026		Vorlesungsfreie Woche
13	23	05.06.2026	UW-BHW-414-09/10	KI, Maschinelles Lernen, Neuronale Netzwerke
15	24	12.06.2026	UW-BHW-414-11/12	Kontinuumsmechanik, Hydromechanik
17	25	19.06.2026	UW-BHW-414-I	Differentialgleichungen, Näherungsverfahren
19	26	26.06.2026	UW-BHW-414-J	Finite-Differenzen, explizite Verfahren
21	27	03.07.2026	UW-BHW-414-K	Finite-Differenzen, implizite Verfahren
23	28	10.07.2026	UW-BHW-414-L	Gerinnehydraulik, Grundwasserhydraulik
25	29	17.07.2026	UW-BHW-414-M	Grundwasserhydraulik
27	30	24.07.2026	UW-BHW-414-N	Zusammenfassung, Klausurvorbereitung

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Fahrplan für heute ...

1. UW-BHW-414-11: Grundlagen der Kontinuumsmechanik
   - Semesterplan

2. Übung: Maschinelles Lernen: Lineare Regression

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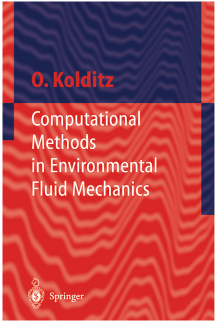
Fahrplan für heute ...

- Motivation
- Lagrange Konzept
- Euler Konzept
- Reynolds Transport Theorem
- Fluxes
- Bilanzgleichungen
- Erhaltungsgrößen



>> Skript

$\Longrightarrow$ Theorie - vor allem die mathematische Schreibweise verstehen "zu lesen"

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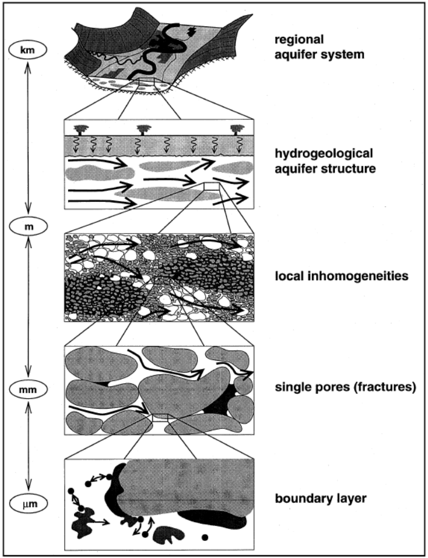
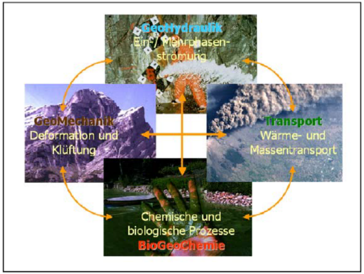
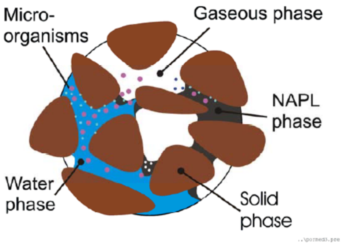
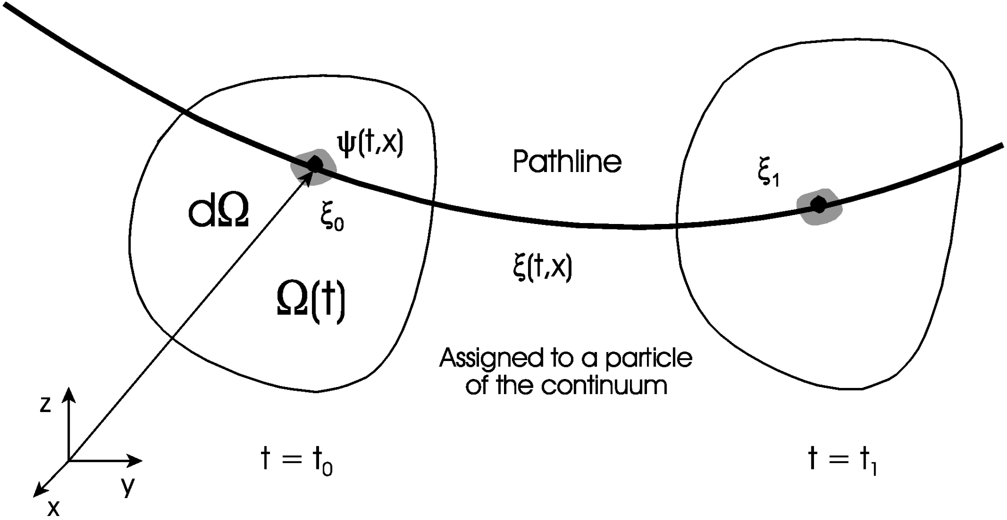
Skalen


km
m
mm
µm
regional
aquifer system
hydrogeological
aquifer structure
local inhomogeneities
single pores (fractures)
boundary layer


GeoHydraulik
Ein-/ Mehrphasen-
strömung
GeoMechanik
Deformation und
Klüftung
Transport
Wärme- und
Massentransport
Chemische und
biologische Prozesse
BioGeoChemie


Micro-
organisms
Gaseous phase
NAPL
phase
Water
phase
Solid
phase



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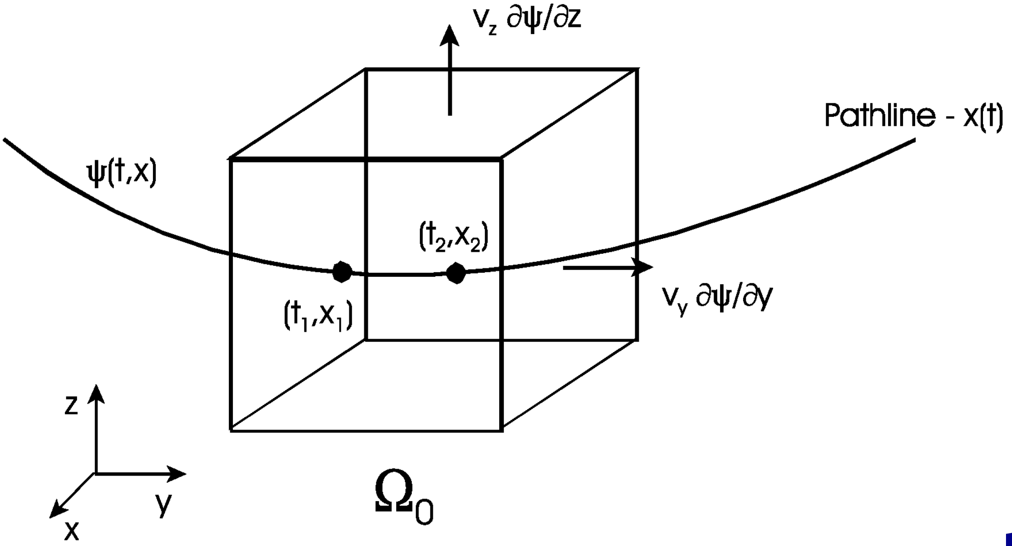
Euler Konzept (1.1.1)



- Volumenintegral (Zeichnung)

$$\int\limits_{\Omega} d\Omega \tag{1}$$

$$\int\limits_{a}^{b} f(x)dx = \lim_{(x_{k+1}-x_k)\rightarrow 0} \quad \sum_{k=1}^{\infty} \left(f(x_{k+1} - f(x_k)\right) \left(x_{k+1} - x_k\right) \tag{2}$$

- Oberflächen-(Ring)-Integral (Zeichnung)

$$\oint\limits_{\partial\Omega} \mathbf{n} \cdot d\mathbf{S} \tag{3}$$

- Materielle Ableitung

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \mathbf{v} \cdot \nabla\psi \tag{4}$$

- Gradient (Vektor)

$$\nabla = \{\partial/\partial x, \partial/\partial y, \partial/\partial z\} \tag{5}$$

- Divergenz (Skalar)

$$\nabla \cdot \mathbf{v} = \partial v_x/\partial x + \partial v_y/\partial y + \partial v_z/\partial z \tag{6}$$

Aufgabe HA01: Was ist $\mathbf{v} \cdot \nabla\psi$?

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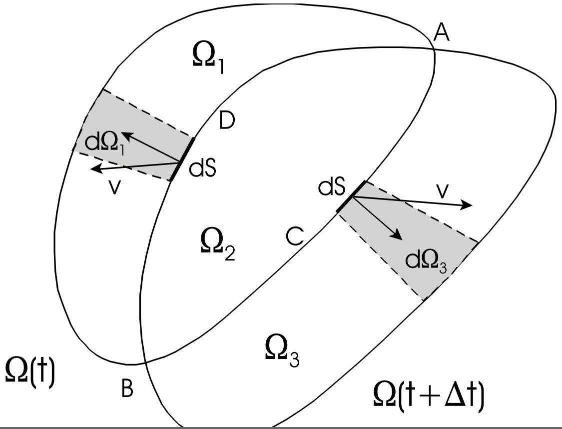
$$\frac{d}{dt}\int\limits_{\Omega}\psi d\Omega = \int\limits_{\Omega}\frac{\partial\psi}{\partial t}d\Omega + \oint\limits_{\partial\Omega}\psi(t)\mathbf{v}\cdot d\mathbf{S} = \int\limits_{\Omega} q^{\psi} d\Omega \qquad (7)$$



Beweisführung: Siehe Skript Abschn. 1.1.3

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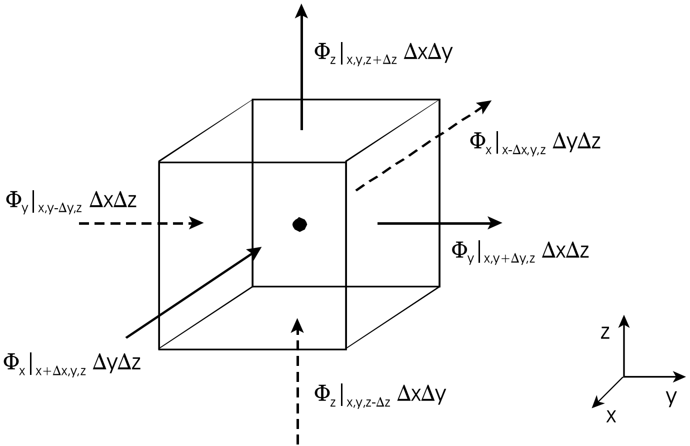
Reynolds Transport Theorem (Euler) (1.1.3)



$$\Phi_x^{\psi} = \partial\psi/\partial x \quad , \quad \Phi^{\psi} = \nabla\psi$$

Frage: Ist Φ^{ψ} eine skalare oder vektorielle Größe ?

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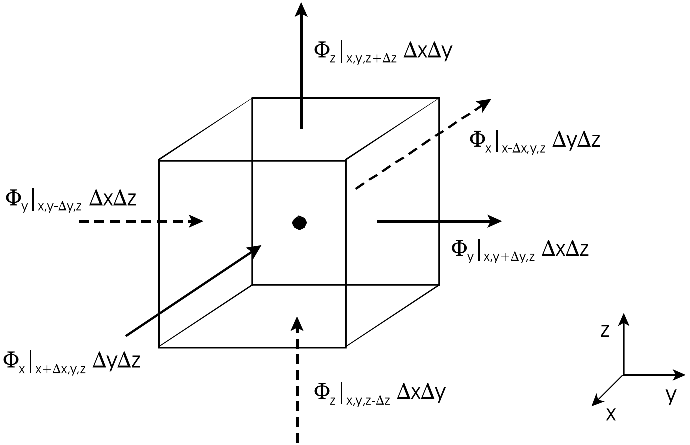
Reynolds Transport Theorem (Euler) (1.1.3)



$$\oint_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega \qquad (8)$$

Reynolds Transport Theorem (Euler) (1.1.3)

Divergence

$$\oint_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega \tag{9}$$

$$\lim_{\Omega\to 0} \frac{1}{\Omega} \oint_{\partial\Omega} \Phi \cdot d\mathbf{S} = \nabla \cdot \Phi \tag{10}$$

Point-Flux

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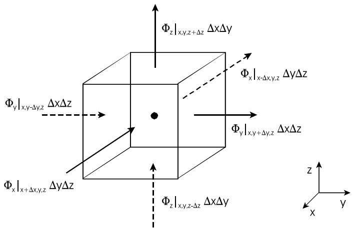
Reynolds Transport Theorem (Euler) (1.1.3)

General Balance Equation



$$\frac{d}{dt}\int_{\Omega}\psi d\Omega = \underbrace{\frac{\partial}{\partial t}\int_{\Omega}\psi d\Omega}_{1} + \underbrace{\oint_{\partial\Omega}\mathbf{\Phi}^{\psi}\cdot d\mathbf{S}}_{2} = \underbrace{\int_{\Omega} q^{\psi} d\Omega}_{3} \qquad (11)$$

with:

1. Rate of change of total amount of quantity ψ in the control volume,
2. Net rate of increase / decrease of ψ due to fluxes,
3. Rate of increase / decrease of ψ due to sources.

Reynolds Transport Theorem (Euler) (1.1.3)

$$\frac{d}{dt}\int\limits_{\Omega} \psi d\Omega = \underbrace{\frac{\partial}{\partial t}\int\limits_{\Omega} \psi d\Omega}_{1} + \underbrace{\oint\limits_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S}}_{2} = \underbrace{\int\limits_{\Omega} q^{\psi} d\Omega}_{3} \tag{12}$$

using

$$\oint\limits_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S} = \int\limits_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega \tag{13}$$

$$\frac{d}{dt}\int\limits_{\Omega} \psi d\Omega = \int\limits_{\Omega} \frac{\partial \psi}{\partial t} d\Omega + \int\limits_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega = \int\limits_{\Omega} q^{\psi} d\Omega \tag{14}$$

Reynolds Transport Theorem

$$\frac{d}{dt}\int\limits_{\Omega}\psi d\Omega = \int\limits_{\Omega}\frac{\partial\psi}{\partial t}d\Omega + \int\limits_{\Omega}\nabla\cdot\Phi^{\psi}d\Omega = \int\limits_{\Omega}q^{\psi}d\Omega \tag{15}$$

$$\forall\Omega : \frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla\cdot\Phi^{\psi} = q^{\psi} \tag{16}$$

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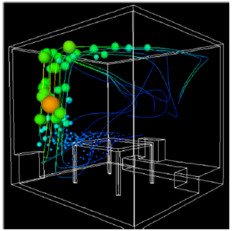
Reynolds Transport Theorem

$$\frac{d}{dt}\int\limits_{\Omega}\psi d\Omega = \int\limits_{\Omega}\left(\frac{\partial\psi}{\partial t} + \nabla\cdot\mathbf{\Phi}^{\psi}\right)d\Omega = \int\limits_{\Omega} q^{\psi} d\Omega$$

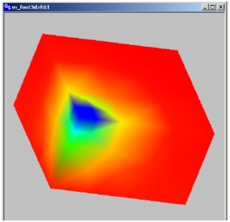
$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \mathbf{v}^{E}\nabla\psi$$

Lagrange

Euler



http://www.cscs.ch/~mvalle/Libro/



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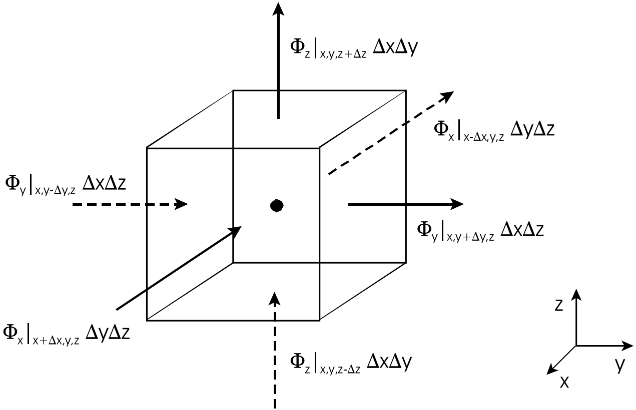
Fluxes (1.1.6)

The total flux Φ^{ψ} of a quantity ψ is defined as

$$\Phi^{\psi} = \mathbf{v}^E \psi \tag{17}$$



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Fluxes (1.1.6)

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla \cdot \Phi^{\psi} = q^{\psi} \tag{18}$$

$$\Phi^{\psi} = \mathbf{v}^{E}\psi \tag{19}$$

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla \cdot (\mathbf{v}^{E}\psi) = q^{\psi} \tag{20}$$

Fluxes (1.1.6)

$$\Phi^{\psi} = \mathbf{v}^E \psi = \underbrace{\mathbf{v}\psi}_{\Phi_A^{\psi}} + \underbrace{(\mathbf{v}^E - \mathbf{v})\psi}_{\Phi_D^{\psi}} \tag{21}$$

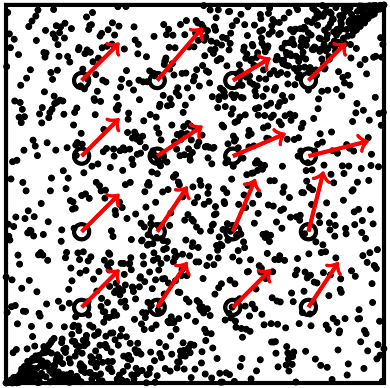
and, therefore, decomposed into two parts: an advective flux Φ_A^{ψ} and a diffusive flux Φ_D^{ψ} relative to the mass-weighted velocity:

- advective flux of quantity ψ

$$\Phi_A^{\psi} = \mathbf{v}\psi \tag{22}$$

- diffusive flux of quantity ψ (Fick's law)

$$\Phi_D^{\psi} = -\mathbf{D}^{\psi} \nabla \psi \tag{23}$$



$$\mathbf{v}^E = \cup\, \mathbf{v}_i$$

$$\mathbf{v}^E = \mathbf{v} + \hat{\mathbf{v}}$$

$$\hat{\mathbf{v}} = \mathbf{v}^E - \mathbf{v}$$

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- Integral form

$$\int_\Omega \frac{d\psi}{dt} = \int_\Omega \frac{\partial \psi}{\partial t} + \int_\Omega \nabla \cdot (\mathbf{v}\psi) - \int_\Omega \nabla \cdot (\mathbf{D}^\psi \nabla \psi) = \int_\Omega Q^\psi \tag{24}$$

- Differential form

$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \nabla \cdot (\mathbf{v}\psi) - \nabla \cdot (\mathbf{D}^\psi \nabla \psi) = Q^\psi \tag{25}$$

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Conservation Quantities (1.1.2)

The amount of a quantity in a defined volume Ω is given by

$$\Psi = \int_{\Omega} \psi d\Omega(t) \tag{26}$$

where Ψ is an extensive conservation quantity (i.e. mass, momentum, energy) and ψ is the corresponding intensive conservation quantity such as mass density ρ, momentum density $\rho\mathbf{v}$ or energy density e.

Extensive quantity	Symbol	Intensive quantity	Symbol
Mass	M	Mass density	ρ
Linear momentum	$\mathbf{m}$	Linear momentum density	$\rho\mathbf{v}$
Energy	E	Energy density	$e = \rho i + \frac{1}{2}\rho v^2$

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Übungen

- Git
- ML: Lineare Regression

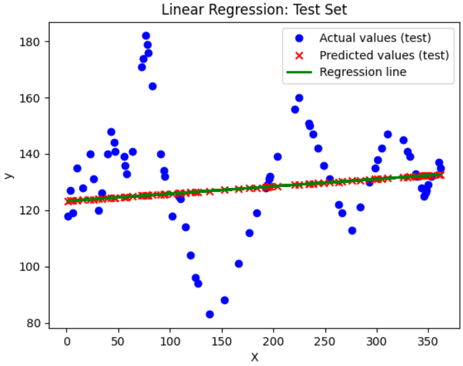

Fig.: water level (River)

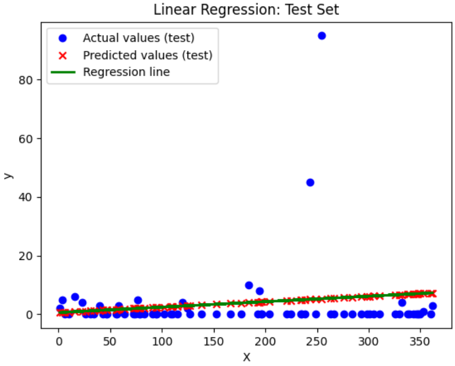

Fig.: precipitation (Shanghai)

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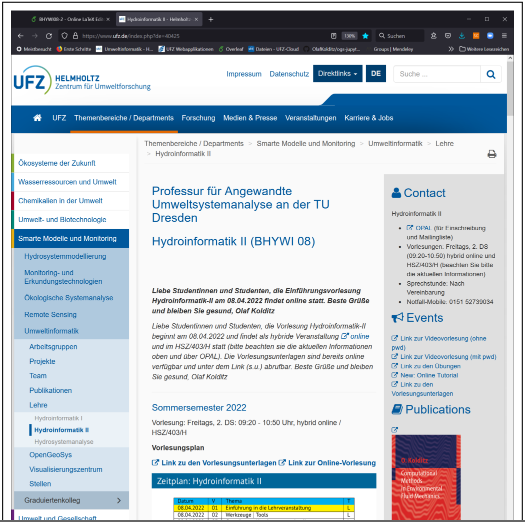
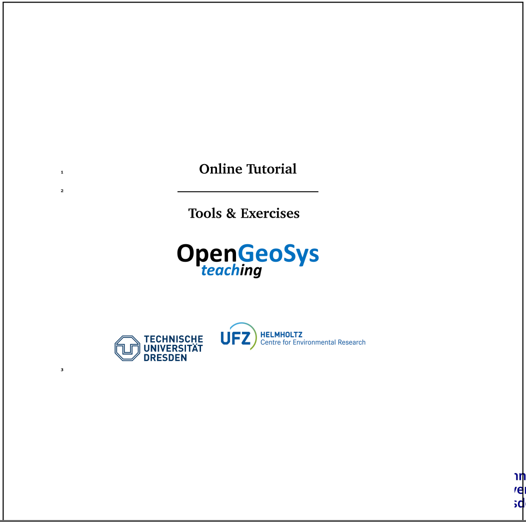
OGS-Teaching Tutorial

Git: Übungen klonen





1.1.3 Cloning sources from a `git` repository

```
Eingabeaufforderung
Microsoft Windows [Version 10.0.18363.1443]
(c) 2019 Microsoft Corporation. Alle Rechte vorbehalten.

C:\Users\okolditz>git clone https://github.com/OlafKolditz/Hydroinformatik-II.git
```

1.1.4 Updating sources from a `git` repository

```
Eingabeaufforderung
C:\Users\okolditz>cd C:\User\15_REP\Hydroinformatik-II

C:\User\15_REP\Hydroinformatik-II>git fetch --all
Fetching origin

C:\User\15_REP\Hydroinformatik-II>git pull
Already up to date.

C:\User\15_REP\Hydroinformatik-II>
```