

# Hydroinformatik - SoSe 2025

## UW-BHW-414-L1: Gerinnehydraulik - Grundlagen

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Dresden, 11.07.2025

# Zeitplan: Hydroinformatik I+II

Sommersemester 2025

Stand: 08.05.2025

| Nr. | KW | Datum      | ID           | Thema                                           |
|-----|----|------------|--------------|-------------------------------------------------|
| 01  | 15 | 11.04.2025 | UW-BHW-414-A | Einführung, Werkzeuge#1, Hello World            |
| 03  | 17 | 25.04.2025 | UW-BHW-414-B | Umweltinformatik, Werkzeuge#2 (git), Datentypen |
| 05  | 18 | 02.05.2025 | UW-BHW-414-C | Selbststudium                                   |
| 07  | 19 | 09.05.2025 | UW-BHW-414-D | Objekt-Orientierte Programmierung: C++, Klassen |
| 09  | 20 | 16.05.2025 | UW-BHW-414-E | Python                                          |
| 11  | 21 | 23.05.2025 | UW-BHW-414-F | Modellierung, Digitalisierung, Wasser 4.0       |
| 13  | 22 | 30.05.2025 | UW-BHW-414-G | KI, Maschinelles Lernen, Neuronale Netzwerke    |
| 15  | 23 | 06.06.2025 | UW-BHW-414-H | Kontinuumsmechanik, Hydromechanik               |
| 05  | 18 | 13.06.2025 |              | Vorlesungsfreie Woche                           |
| 17  | 25 | 20.06.2025 | UW-BHW-414-I | Differentialgleichungen, Näherungsverfahren     |
| 19  | 26 | 27.06.2025 | UW-BHW-414-J | Finite-Differenzen, explizite Verfahren         |
| 21  | 27 | 04.07.2025 | UW-BHW-414-K | Finite-Differenzen, implizite Verfahren         |
| 23  | 28 | 11.07.2025 | UW-BHW-414-L | Gerinnehydraulik, Grundwasserhydraulik          |
| 25  | 29 | 18.07.2025 | UW-BHW-414-M | Zusammenfassung, Klausurvorbereitung            |

- 1 Abfallwirtschaft / Wärmeversorgung: Diffusionsprozesse
- 2 **Hydrology: Gerinnehydraulik (→ this)**
- 3 Grundwasserwirtschaft: Grundwasserhydraulik (→ next)

## 0 Finite-Differenzen Verfahren (explizit und implizit)

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1 Gerinnehydraulik: Parameter, Symbole ...

2 Energieerhaltung: Bernoulli-Gleichung

3 Impulserhaltung: Saint-Venant-Gleichungen

4 Lösung der Bernoulli-Gleichung

5 Newton-Verfahren

6  $\mapsto$  numerisches Lösungsverfahren (nächste Veranstaltung)

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7 Abschluss Übungen

8 Klausurvorbereitung

# Hydrologie - Gerinnehydraulik

Für die Beschreibung von Strömungsprozessen in Flüssen (d.h. Gerinne) gibt es zwei verschiedene Möglichkeiten basierend auf den sogenannten Bernoulli (Energiebetrachtung) oder Saint-Venant Gleichungen. Wir verwenden die erste für die Implementierung in C++ (Bildquellen: Technische Hydromechanik).



- ▶ History
- ▶ gelebt 1700-1782
- ▶ Freund von L Euler
- ▶ Bernoulli Gleichung
- ▶ kinetische Theorie der Gase
- ▶ russ. Akademie Petersburg
- ▶ Vater/Sohn Beziehung

$$z_{i+1} + \frac{p_{i+1}}{\rho g} + \frac{v_{i+1}^2}{2g} = z_i + \frac{p_i}{\rho g} + \frac{v_i^2}{2g} + h_f \quad (1)$$

- ▶ Symbole
- ▶ Gesetz von Darcy-Weisbach für streckenabhängige Verluste

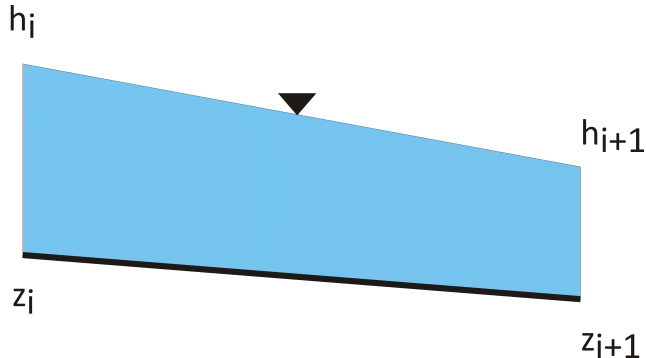
$$h_f = \lambda \frac{x_{i+1} - x_i}{d_{hy}} \frac{v^2}{2g} \quad (2)$$

- ▶ Symbole (Zusammenfassung später)
- ▶ Gesetz von Darcy-Weisbach für streckenabhängige Verluste

$$d_{hy} = 4r_{hy} = 4\frac{A}{p}$$

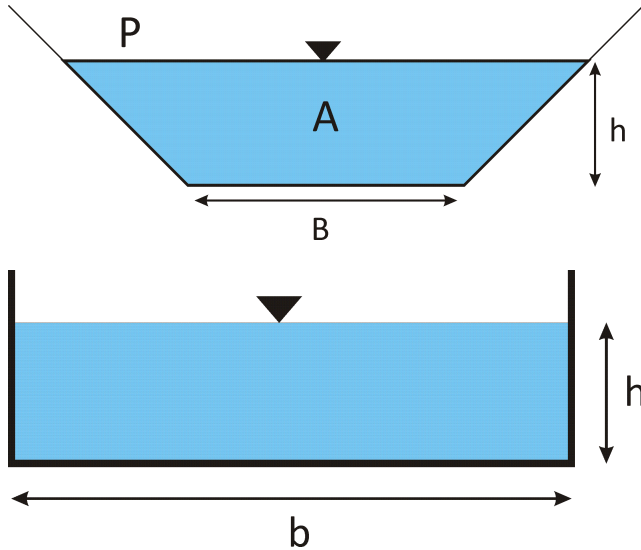
$$p = \rho gh \quad (4)$$

$$z_{i+1} + h_{i+1} + \frac{v_{i+1}^2}{2g} = z_i + h_i + \frac{v_i^2}{2g} + h_f, E = h + \frac{v^2}{2g} \quad (5)$$





# Channel Geometry



- Durchfluss

$$Q = A v = b h v \quad (6)$$

$$z_{i+1} + h_{i+1} + \frac{Q_{i+1}^2}{2g b^2 h_{i+1}^2} = z_i + h_i + \frac{Q_i^2}{2g b^2 h_i^2} + h_f \quad (7)$$

# Fluid Momentum Balance - Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{f}^e + \frac{1}{\rho} \nabla \cdot \sigma \quad (8)$$

In index notation the above vector equation is written as

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= \frac{1}{\rho} \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} \right) \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= \frac{1}{\rho} \left( \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} \right) \\ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= g + \frac{1}{\rho} \left( \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \right) \end{aligned} \quad (9)$$

with  $u = v_x, v = v_y, w = v_z$  and  $\mathbf{f}^e = \mathbf{g}$ .

- ▶ History
- ▶ gelebt 1797-1886
- ▶ 1D open channel flow
- ▶ shallow water equations (Flachwassergleichungen)
- ▶ Stokes equation (is his)
- ▶ Engineer
- ▶ Count

# Saint-Venant-Gleichungen #1

- ▶ Saint-Venant-Gleichungen
- ▶ Vertikal gemittelte Navier-Stokes-Gleichung
- ▶ 2D Flachwasser-Gleichungen (Filmchen)
- ▶ Mittelung über die Querschnittsfläche
- ▶ 1D Gerinnehydraulik
- ▶ Fließgeschwindigkeit  $v = Q/A$  in Flußrichtung

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad (10)$$

$$\frac{\partial Q}{\partial t} \frac{1}{A} + \frac{\partial}{\partial x} \left( \frac{Q^2}{2A^2} + gh(x, A) \right) = g(S_o - S_f) \quad (11)$$

- ▶ Stationäre Bedingungen

$$\begin{aligned}\frac{d}{dx} (A(x, h)v) &= 0 \\ \frac{d}{dx} \left( \frac{v^2}{2} + gh(x, A) \right) &= g(S_o - S_f)\end{aligned}\tag{12}$$

unter Verwendung der Primärvariablen  $A$  und  $Q$

$$\frac{d}{dx} \left( \frac{Q(h)^2}{2A(h)^2} + gh(x, A) \right) = g(S_o - S_f)\tag{13}$$

- ▶ Nichtlineare Gleichung

| Symbol      | Parameter                | Englisch                   | Einheit                   |
|-------------|--------------------------|----------------------------|---------------------------|
| $h$         | Wasserspiegel            | water surface elevation    | m                         |
| $i$         | Sohlenspiegel            | channel datum              | m                         |
| $z$         | Wassertiefe              | water depth                | m                         |
| $Z_L, Z_R$  | linke / rechte Böschung  | left / right channel slope | m                         |
| $Q$         | Durchfluss               | discharge                  | $\text{m}^3\text{s}^{-1}$ |
| $A$         | Querschnittsfläche       | channel cross-section      | $\text{m}^2$              |
| $B$         | Breite                   | channel width              | $\text{m}^2$              |
| $P$         | benetzter Umfang         | wetted perimeter           | m                         |
| $R, r_{hy}$ | hydraulische Radius      | hydraulic radius           | m                         |
| $U$         | stromaufwärts            | upstream                   | -                         |
| $D$         | stromabwärts             | downstream                 | -                         |
| $\Delta x$  | Abstand zwischen U und D | distance between U and D   | m                         |
| $h_v$       | kinetische Energie       | velocity head              | m                         |
| $h_f$       | Reibungsverluste         | friction head              | m                         |
| $S_o$       | Sohlgefälle              | bed slope                  | -                         |
| $S_f$       | Streckenverlust          | friction slope             | -                         |

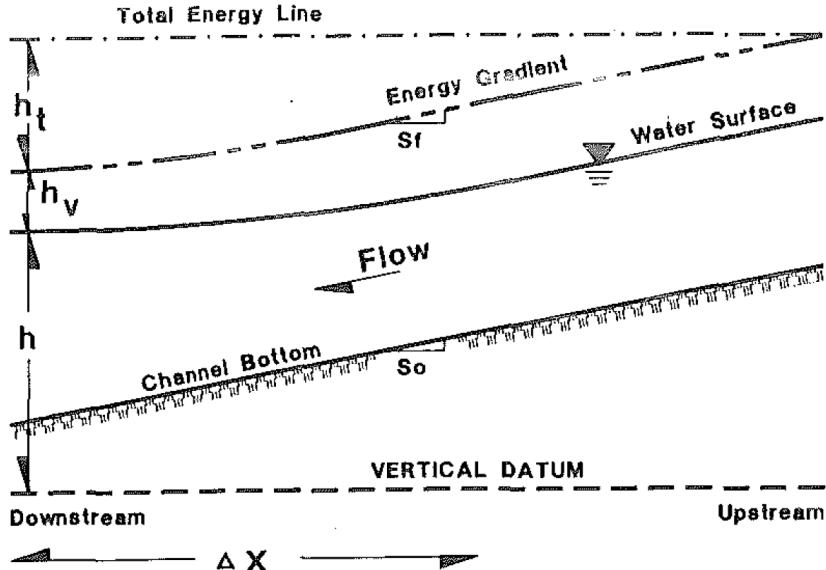
$$z_{i+1} + \frac{p_{i+1}}{\rho g} + \frac{v_{i+1}^2}{2g} = z_i + \frac{p_i}{\rho g} + \frac{v_i^2}{2g} + h_f \quad (14)$$

$$h_f = \lambda \frac{x_{i+1} - x_i}{d_{hy}} \frac{v^2}{2g} \quad (15)$$

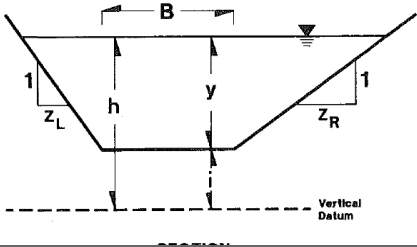
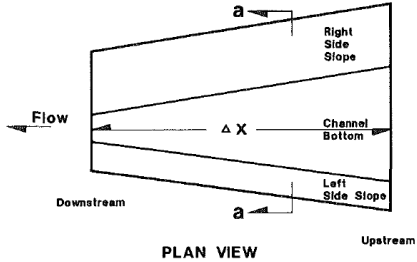
$$d_{hy} = 4r_{hy} = 4 \frac{A}{p} \quad (16)$$



# Energiebetrachtung #2: Bernoulli/Saint-Venant



## Energiebetrachtung #3: Bernoulli



# Energiebetrachtung #4 Geometrie

- Die trapezoide Querschnittsfläche des Gerinnes kann folgendermaßen berechnet werden (Abb. 2).

$$A = y \left[ B + \frac{Z_L + Z_R}{2} \right] \quad (17)$$

- Der benetzte Umfang des Gerinnes ist.

$$P = B + y \left[ \sqrt{1 + Z_L^2} + \sqrt{1 + Z_R^2} \right] \quad (18)$$

- Der hydraulische Radius ergibt sich dann als:

$$R = \frac{A}{P} \quad (19)$$

- Letztlich benötigen wir noch den Zusammenhang von Fließgeschwindigkeit und Durchfluss.

$$v = \frac{Q}{A} = \frac{Q}{y \left[ B + \frac{Z_L + Z_R}{2} \right]}$$

Die Streckenverluste lassen sich nach Manning wie folgt beschreiben

$$S_f = \left( \frac{Q}{AR^{2/3}} \right)^2 \quad (21)$$

$$S_f = Q^2 A^{-2} R^{-4/3} \quad (22)$$

mit

$$R = \frac{y(B + C_4)}{B + yC_5} \quad (23)$$

$$C_4 = \frac{Z_L + Z_R}{2} \quad (24)$$

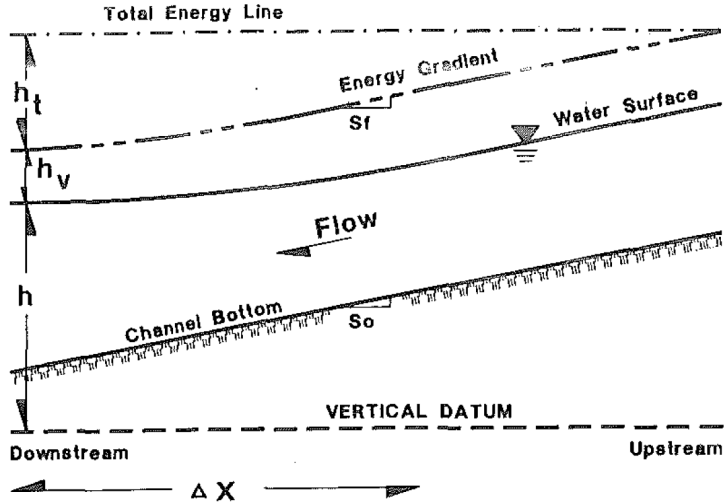
$$C_5 = \sqrt{1 + Z_L^2} + \sqrt{1 + Z_R^2} \quad (25)$$

$$S_f = Q^2 (y(B + C_4))^{-2} \left( \frac{y(B + C_4)}{B + yC_5} \right)^{-4/3} \quad (26)$$

$$S_f = Q^2 (By + C_4y)^{-2} (By + C_4y)^{-4/3} (B + C_5y)^{4/3} \quad (27)$$

$$S_f = Q^2 (By + C_4y)^{-10/3} (B + C_5y)^{4/3}$$

# Lösung einer nichtlinearen Gleichung #1



$$f(h) = \left( h + \frac{v^2}{2g} \right) |_D - \left( h + \frac{v^2}{2g} \right) |_U + \Delta x \frac{(S_{f,U} + S_{f,D})}{2} \quad (30)$$

$$f(h) = \left( h + \frac{Q^2}{2gA^2} \right) |_D - \left( h + \frac{Q^2}{2gA^2} \right) |_U + \Delta x \frac{(S_{f,U} + S_{f,D})}{2} \quad (31)$$

- Konstruktion eines Newton-Verfahrens

$$h_{k+1} = h_k + \frac{f(h_k)}{f'(h_k)} \quad (32)$$

- ▶ Nichtlineare Gleichung

$$\mathbf{A}(\mathbf{x}) \mathbf{x} = \mathbf{b}(\mathbf{x}) \quad (f(h) = 0) \quad (33)$$

- ▶ Residuum

$$\mathbf{R}(\mathbf{x}) = \mathbf{A}(\mathbf{x}) \mathbf{x} - \mathbf{b}(\mathbf{x}) = 0 \quad (34)$$

- ▶ Taylor-Reihe

$$\mathbf{R}_{k+1} = \mathbf{R}_k + \left[ \frac{\partial \mathbf{R}}{\partial \mathbf{x}} \right]_k \Delta \mathbf{x}_{k+1} + 0(\Delta \mathbf{x}_{k+1}^2) \quad (35)$$

- ▶ Umstellen

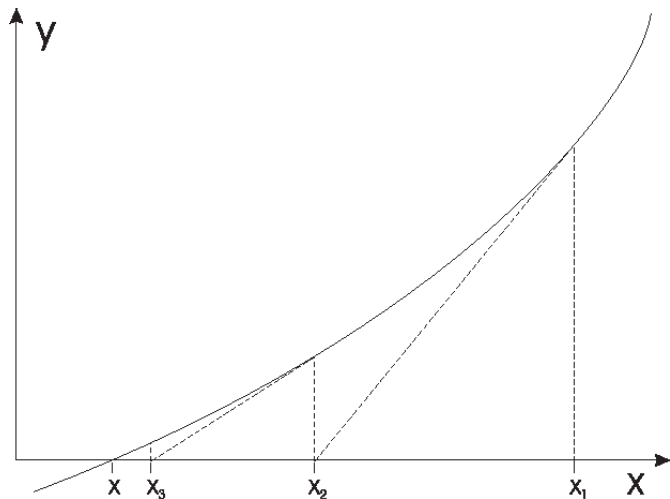
$$\Delta \mathbf{x}_{k+1} \approx -\mathbf{J}_k^{-1} \mathbf{R}_k \quad (\mathbf{R}_{k+1} = 0) \quad (36)$$

- ▶ Lösungsvorschrift

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta \mathbf{x}_{k+1} = \mathbf{x}_k - \mathbf{J}_k^{-1} \mathbf{R}_k$$



# Math: Newton-Verfahren #2



$$y = R(x)$$

- ▶ Newton-Verfahren:  $\mathbf{x} = h$

$$h_{k+1} = h_k + \frac{f(h_k)}{f'(h_k)} = \frac{\mathbf{R}_k}{\mathbf{J}_k} \quad (38)$$

- ▶  $f'(h) = f'(y)$

$$\frac{d}{dh} \left( h + \frac{v^2}{2g} \right) = \frac{d}{dh} \left( h + \frac{Q^2}{2gA^2} \right) = 1 - \frac{Q^2}{2gA^3} \frac{dA}{dh} \quad (39)$$

$$\frac{dA}{dh} = \frac{d}{dh} (y(B + C_4)) = B + C_4 \quad (40)$$

Bleibt noch die Differenzierung der Streckenverluste

$$\frac{dS_f}{dh} = S'_f = \frac{d}{dh} \left( \frac{Q}{AR^{2/3}} \right)^2 \quad (41)$$

$$\begin{aligned} S'_f = & \left[ Q^2 (By + C_4 y)^{10/3} \frac{4}{3} (B + C_5 y)^{1/3} C_5 \right] \\ & + \left[ (B + y C_5)^{4/3} \frac{-10 Q^2}{3} (By + C_4 y)^{13/3} (B + C_4) \right] \end{aligned} \quad (42)$$

$$\begin{aligned} S'_f = & \frac{4}{3} Q^2 C_5 (By + C_4 y)^{-10/3} (B + C_5 y)^{1/3} \\ & - \frac{10}{3} Q^2 (B + C_4) (B + C_5 y)^{4/3} (By + C_4 y)^{-13/3} \end{aligned} \quad (43)$$

# Hydroinformatik - SoSe 2025

## UW-BHW-414-L2: Gerinnehydraulik - Übungen

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Dresden, 11.07.2025

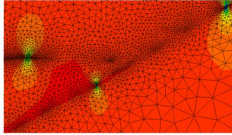
# Zeitplan: Hydroinformatik I+II

Sommersemester 2025

Stand: 08.05.2025

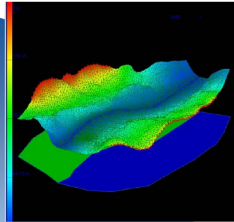
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$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \mathbf{v}^E \nabla \psi$$

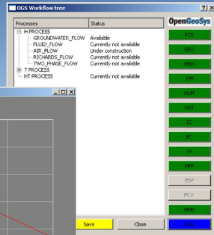


Basics  
Mechanik

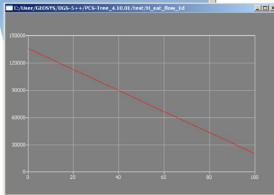
Anwendung



Numerische  
Methoden



Programmierung  
Visual C++



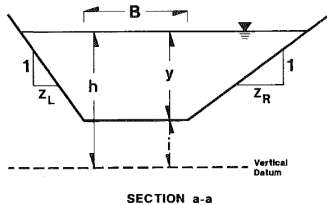
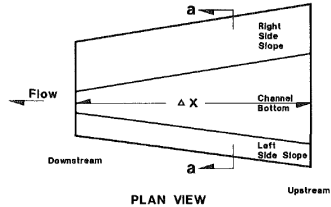
Prozessverständnis

- 1 Abfallwirtschaft: Diffusionsprozesse
- 2 **Hydrology: Gerinnehydraulik (→ this)**
- 3 Grundwasserwirtschaft: Grundwasserhydraulik (→ next)

# Wiederholung: BHYWI-08-12 Gerinnehydraulik - Grundlagen



# Energiebetrachtung #3: Bernoulli



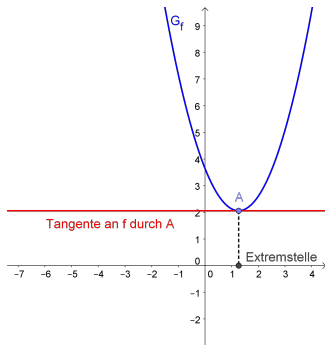
$$y \equiv z$$

Fig.: Trapezgerinne Paine (1992)

# Newton-Verfahren #2

Funktional: Energieerhaltung (potentielle + kinetische + Wärme-Energie (Reibung))

$$f(h) = \left( h + \frac{Q^2}{2gA^2} \right) |_{i+1} - \left( h + \frac{Q^2}{2gA^2} \right) |_i + (x_{i+1} - x_i) \frac{S_{f,i+1} + S_{f,i}}{2} \quad (1)$$



$$\begin{aligned} f'(h) &= \frac{df}{dh} = 0 \\ &\approx \frac{f(h_{k+1}) - f(h_k)}{h_{k+1} - h_k} \\ h_{k+1} &= h_k - \frac{f(h_k)}{f'(h)} = h_k - \frac{N}{D} \end{aligned} \quad (2)$$

Quelle: [https://menzelths.github.io/Mathematik-Kursstufe/Ableitungen\\_Extremstellen.html](https://menzelths.github.io/Mathematik-Kursstufe/Ableitungen_Extremstellen.html)

$$f(h) = \underbrace{\left(h + \frac{Q^2}{2gA^2}\right) \Big|_{i+1}}_{N_1} - \underbrace{\left(h + \frac{Q^2}{2gA^2}\right) \Big|_i}_{N_2} + \underbrace{(x_{i+1} - x_i) \frac{S_{f,i+1} + S_{f,i}}{2}}_{N_3} \quad (3)$$

$$f'(h) = \underbrace{\frac{d}{dh} \left(h + \frac{Q^2}{2gA^2}\right) \Big|_{i+1}}_{D_1} - \underbrace{\frac{d}{dh} \left(h + \frac{Q^2}{2gA^2}\right) \Big|_i}_{D_2} + \underbrace{\frac{x_{i+1} - x_i}{2} \frac{d}{dh} (S_{f,i+1} + S_{f,i})}_{D_3} \quad (4)$$

$$D_1 = \frac{d}{dh} \left( h + \frac{Q^2}{2gA^2} \right) = 1 - \frac{Q^2}{gA^3} \frac{dA}{dh} \quad (5)$$

$$\frac{dA}{dh} = \frac{d}{dh} (y(B + yC_4)) = B + 2yC_4 \quad (6)$$

$$D_1 = 1 - \frac{Q^2}{gA^3} B + 2yC_4 \quad (7)$$

Bleibt noch die Differenzierung der Streckenverluste

$$D_3 = \frac{dS_f}{dh} = S'_f = \frac{d}{dh} \left( \frac{Q}{AR^{2/3}} \right)^2 \quad (8)$$

$$\frac{d}{dh} \left( \frac{Q}{AR^{2/3}} \right)^2 = Q^2 \frac{d}{dh} \left( A^{-2} R^{-4/3} \right) \quad (9)$$

$$\dots = Q^2 \left( \frac{dA^{-2}}{dh} R^{-4/3} \right) + \left( A^{-2} \frac{dR^{-4/3}}{dh} \right) \quad (10)$$

$$\dots = Q^2 \left( -2A^{-3} \frac{dA}{dh} R^{-4/3} \right) + \left( A^{-2} \frac{(-4)}{3} R^{-7/3} \frac{dR}{dh} \right) \quad (11)$$

Bleibt noch die Differenzierung der Streckenverluste

$$\frac{dS_f}{dh} = S'_f = \frac{d}{dh} \left( \frac{Q}{AR^{2/3}} \right)^2 \quad (12)$$

$$\begin{aligned} S'_f = & \left[ Q^2 (By + C_4 y^2)^{10/3} \frac{4}{3} (B + C_5 y)^{1/3} C_5 \right] \\ & + \left[ (B + yC_5)^{4/3} \frac{-10Q^2}{3} (By + C_4 y^2)^{13/3} (B + 2C_4 y) \right] \end{aligned} \quad (13)$$

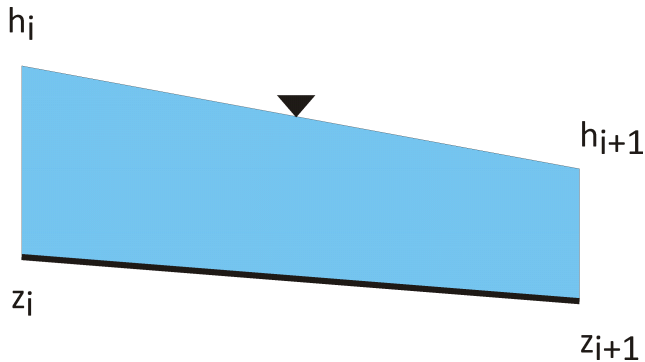
$$\begin{aligned} S'_f = & \frac{4}{3} Q^2 C_5 (By + C_4 y^2)^{-10/3} (B + C_5 y)^{1/3} \\ & - \frac{10}{3} Q^2 (B + 2C_4 y) (B + C_5 y)^{4/3} (By + C_4 y^2)^{-13/3} \end{aligned} \quad (14)$$

# Übungen

## BHYWI-08-11

### Gerinnehydraulik

- ▶ **Q&D:** Übung 1  
(BHYWI-08-05-E):  
Funktionalität
- ▶ **OOP:** Übung 2  
(BHYWI-08-06-E):  
Modularität
- ▶ **GUI:** Übung 3  
(BHYWI-08-07-E): Interaktion  
(Ausgabe)
- ▶ **GUI:** Übung 4  
(BHYWI-08-08-E): Interaktion  
(Eingabe)



BHYWI-08-05-E > BHYWI-08-11A

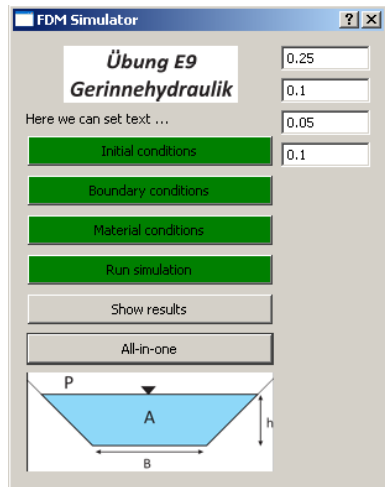
BHYWI-08-06-E > BHYWI-08-11B

BHYWI-08-07-E > BHYWI-08-11C

BHYWI-08-08-E > BHYWI-08-11D



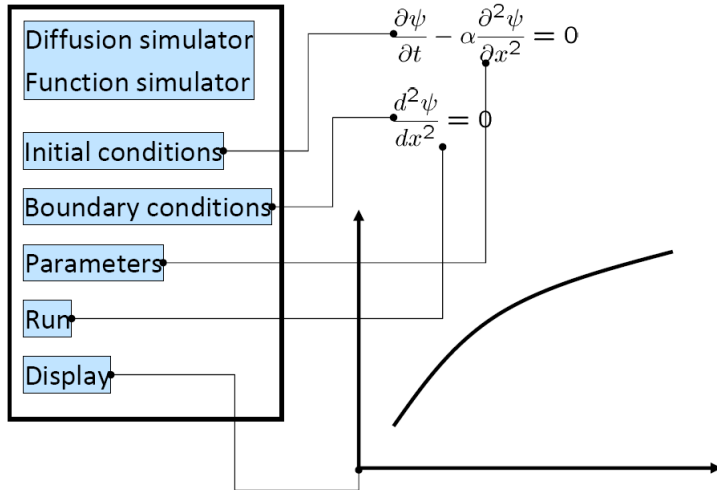
- ▶ **Q&D:** Übung 1  
(BHYWI-08-05-E):  
Funktionalität
- ▶ **OOP:** Übung 2  
(BHYWI-08-06-E):  
Modularität
- ▶ **GUI:** Übung 3  
(BHYWI-08-07-E): Interaktion  
(Ausgabe)
- ▶ **GUI:** Übung 4  
(BHYWI-08-08-E): Interaktion  
(Eingabe)



# Übungen

## BHYWI-08-11A-D

### Gerinnehydraulik



```
1 int main(int argc, char *argv[])
2 {
3     // 1-Geometrie
4     // 2-Anfangsbedingungen
5     // 3-Randbedingungen
6     // 4-Parameter
7     // 5-Berechnungsgroessen
8     // 6-Berechnung (Newton-Verfahren)
9     // 7-Ausgabe der Ergebnisse
10    // File
11    // x-y Plot
12 }
13
14 class channel (:: geo, ic, bc, mat, run, out)
```

**Listing:** Programmstruktur und OOP

$$h_{k+1} = h_k - \frac{f(h_k)}{f'(h_k)} = h_k - \frac{N}{D} \quad (15)$$

```
1 void channel::newton-step
2 {...
3     double N,N1,N2,N3,D,D1,D2,D21,D22;
4     for(int i=0;i<n-1;i++)
5     {
6         N1 = pow(discharge,2)/pow(wetted_cross_section[i+1],2) + gravity*u_old[i+1];
7         N2 = pow(discharge,2)/pow(wetted_cross_section[i],2) + gravity*u_old[i];
8         N3 = gravity*(bed_slope - (friction_slope[i+1]+friction_slope[i])/2.)*(x[i+1]-x[i]);
9         N = N1 - N2 - N3;
10        D1 = pow(discharge,2)/pow(wetted_cross_section[i],3) * (bottom_width+2.*m*u_old[i]) - gravity;
11        D21 = friction_law_exponent*2.*(sqrt(1+m*m))/wetted_perimeter[i];
12        D22 = (1.+friction_law_exponent)/wetted_cross_section[i] * (bottom_width+2.*m*u_old[i]);
13        D2 = gravity*friction_slope[i]*(D21-D22)*(x[i+1]-x[i]);
14        D = D1 + D2;
15        u_new[i] = u_old[i] - N/D;
16    }
17    ...
18 }
```

Listing: Newton-Schritt

# Definition der konstanten Modellparameter

BHYWI-08-11A

```
1 // Parameter
2 double discharge = 0.05; // Volumenfließrate [m3/s]
3 double gravity = 9.81; // [m/s2]
4 double friction_law_exponent = 0.5; // Chezy, Manning-Strickler [-]
5 double error_tolerance = 1e-3; // [m]
6 double bed_slope = 0.0004; // [m/m]
7 double bottom_width = 1.; // [m]
8 double m = 1.; //
9 double friction_coefficient = 10.; //
```

# Definition der funktionalen Modellparameter

BHYWI-08-11A

```
1 // Berechnungsgroessen
2 // Newton-Verfahren
3 double wetted_perimeter[n];
4 double wetted_cross_section[n];
5 double water_level_elevation[n];
6 // Abgeleitete Groessen (Ausgabe)
7 double hydraulic_radius[n];
8 double flow_velocity[n];
9 double Froude_number[n];
10 double friction_slope[n];
```

# Berechnung der funktionalen Modellparameter

BHYWI-08-11A

```
1  for(int i=0;i<n;i++)
2  {
3      wetted_perimeter[i] = bottom_width + 2.*sqrt(1.+m*m)*u_old[i];
4      wetted_cross_section[i] = (bottom_width + m*u_old[i])*u_old[i];
5      hydraulic_radius[i] = wetted_cross_section[i] / wetted_perimeter[i];
6      water_level_elevation[i] = bottom_elevation[i] + u_old[i];
7      flow_velocity[i] = discharge/wetted_cross_section[i];
8      Froude_number[i] = flow_velocity[i]/(sqrt(gravity*
9      wetted_cross_section[i]\
10      /sqrt(bottom_width*bottom_width+4.*m*
11      wetted_cross_section[i])));
12      friction_slope[i] = pow(flow_velocity[i]/(friction_coefficient*
13      pow(hydraulic_radius[i],friction_law_exponent))
14      ,2);
15  }
```



```
1 void channel::out
2 {...
3   ofstream out_file("out.txt");
4   out_file.precision(4);
5   out_file << "Water depth (old):\t";
6   for(int i=0;i<n;i++)
7   {
8     out_file << "\t" << u_old[i] << " ";
9   }
10  out_file << endl;
11  ...
12  out_file << "Water depth (new):\t";
13  for(int i=0;i<n;i++)
14  {
15    out_file << "\t" << u_new[i] << " ";
16  }
17  out_file << endl;
18  out_file.close();
19  ...}
20 }
```

Listing: Test\_Ausgabe

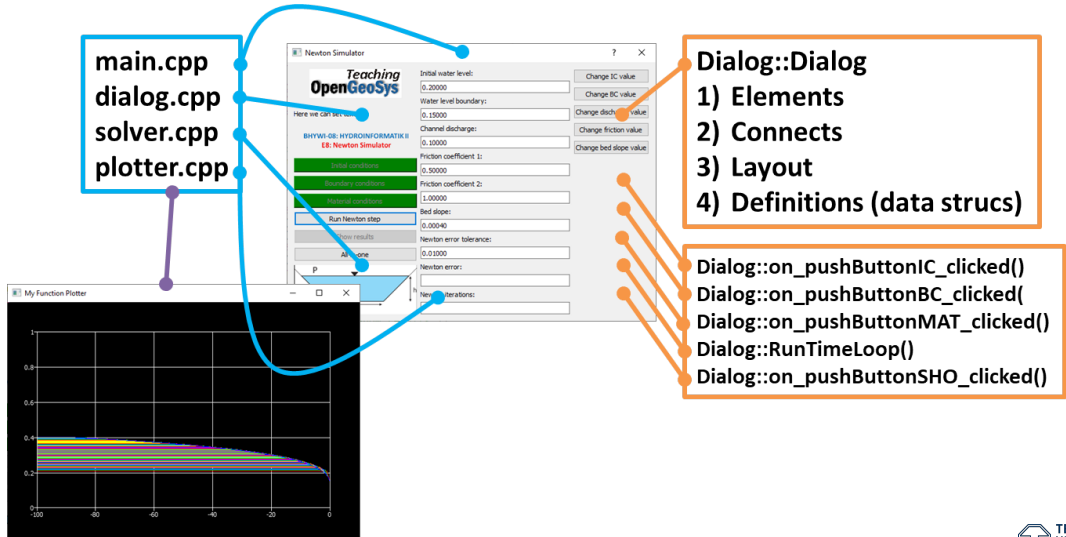
| out.txt - Editor                  |         |        |         |        |         |        |         |        |         |        |         |        |
|-----------------------------------|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|
| Datei Bearbeiten Format Ansicht ? |         |        |         |        |         |        |         |        |         |        |         |        |
| 0.25                              |         |        |         |        |         |        |         |        |         |        |         |        |
| 0.1                               |         |        |         |        |         |        |         |        |         |        |         |        |
| Iteration: 0                      |         |        |         |        |         |        |         |        |         |        |         |        |
| water depth (old):                |         | 0.25   | 0.25    | 0.25   | 0.25    | 0.25   | 0.25    | 0.25   | 0.25    | 0.25   | 0.25    | 0.1    |
| wetted perimeter:                 |         | 1.71   | 1.71    | 1.71   | 1.71    | 1.71   | 1.71    | 1.71   | 1.71    | 1.71   | 1.71    | 1.28   |
| wetted cross section:             | 0.313   | 0.313  | 0.313   | 0.313  | 0.313   | 0.313  | 0.313   | 0.313  | 0.313   | 0.313  | 0.313   | 0.11   |
| Hydraulic radius:                 |         | 0.183  | 0.183   | 0.183  | 0.183   | 0.183  | 0.183   | 0.183  | 0.183   | 0.183  | 0.183   | 0.0857 |
| water level elevation:            | 0.29    | 0.286  | 0.282   | 0.278  | 0.274   | 0.27   | 0.266   | 0.262  | 0.258   | 0.254  | 0.1     |        |
| Flow velocity:                    | 0.16    | 0.16   | 0.16    | 0.16   | 0.16    | 0.16   | 0.16    | 0.16   | 0.16    | 0.16   | 0.455   |        |
| Froude number:                    | 0.112   | 0.112  | 0.112   | 0.112  | 0.112   | 0.112  | 0.112   | 0.112  | 0.112   | 0.112  | 0.479   |        |
| Friction slope:                   | 0.0014  | 0.0014 | 0.0014  | 0.0014 | 0.0014  | 0.0014 | 0.0014  | 0.0014 | 0.0014  | 0.0014 | 0.0241  |        |
| water depth (new):                |         | 0.259  | 0.259   | 0.259  | 0.259   | 0.259  | 0.259   | 0.259  | 0.259   | 0.259  | 0.242   | 0.1    |
| Iteration: 1                      |         |        |         |        |         |        |         |        |         |        |         |        |
| water depth (old):                |         | 0.259  | 0.259   | 0.259  | 0.259   | 0.259  | 0.259   | 0.259  | 0.259   | 0.259  | 0.242   | 0.1    |
| wetted perimeter:                 |         | 1.73   | 1.73    | 1.73   | 1.73    | 1.73   | 1.73    | 1.73   | 1.73    | 1.73   | 1.69    | 1.28   |
| wetted cross section:             | 0.326   | 0.326  | 0.326   | 0.326  | 0.326   | 0.326  | 0.326   | 0.326  | 0.326   | 0.301  | 0.11    |        |
| Hydraulic radius:                 |         | 0.188  | 0.188   | 0.188  | 0.188   | 0.188  | 0.188   | 0.188  | 0.188   | 0.188  | 0.179   | 0.0857 |
| water level elevation:            | 0.299   | 0.295  | 0.291   | 0.287  | 0.283   | 0.279  | 0.275   | 0.271  | 0.267   | 0.246  | 0.1     |        |
| Flow velocity:                    | 0.153   | 0.153  | 0.153   | 0.153  | 0.153   | 0.153  | 0.153   | 0.153  | 0.153   | 0.166  | 0.455   |        |
| Froude number:                    | 0.105   | 0.105  | 0.105   | 0.105  | 0.105   | 0.105  | 0.105   | 0.105  | 0.105   | 0.118  | 0.479   |        |
| Friction slope:                   | 0.00125 |        | 0.00125 |        | 0.00125 |        | 0.00125 |        | 0.00125 |        | 0.00125 |        |
| water depth (new):                |         | 0.267  | 0.267   | 0.267  | 0.267   | 0.267  | 0.267   | 0.267  | 0.267   | 0.253  | 0.242   | 0.1    |
| Iteration: 2                      |         |        |         |        |         |        |         |        |         |        |         |        |
| water depth (old):                |         | 0.267  | 0.267   | 0.267  | 0.267   | 0.267  | 0.267   | 0.267  | 0.267   | 0.253  | 0.242   | 0.1    |
| wetted perimeter:                 |         | 1.76   | 1.76    | 1.76   | 1.76    | 1.76   | 1.76    | 1.76   | 1.76    | 1.72   | 1.69    | 1.28   |
| wetted cross section:             | 0.339   | 0.339  | 0.339   | 0.339  | 0.339   | 0.339  | 0.339   | 0.339  | 0.317   | 0.301  | 0.11    |        |
| Hydraulic radius:                 |         | 0.193  | 0.193   | 0.193  | 0.193   | 0.193  | 0.193   | 0.193  | 0.193   | 0.185  | 0.179   | 0.0857 |
| water level elevation:            | 0.307   | 0.303  | 0.299   | 0.295  | 0.291   | 0.287  | 0.283   | 0.279  | 0.261   | 0.246  | 0.1     |        |
| Flow velocity:                    | 0.148   | 0.148  | 0.148   | 0.148  | 0.148   | 0.148  | 0.148   | 0.148  | 0.158   | 0.166  | 0.455   |        |
| Froude number:                    | 0.1     | 0.1    | 0.1     | 0.1    | 0.1     | 0.1    | 0.1     | 0.1    | 0.11    | 0.118  | 0.479   |        |
| Friction slope:                   | 0.00113 |        | 0.00113 |        | 0.00113 |        | 0.00113 |        | 0.00113 |        | 0.00113 |        |
| water depth (new):                |         | 0.274  | 0.274   | 0.274  | 0.274   | 0.274  | 0.274   | 0.274  | 0.262   | 0.253  | 0.242   | 0.1    |
| Iteration: 3                      |         |        |         |        |         |        |         |        |         |        |         |        |
| water depth (old):                |         | 0.274  | 0.274   | 0.274  | 0.274   | 0.274  | 0.274   | 0.274  | 0.262   | 0.253  | 0.242   | 0.1    |
| wetted perimeter:                 |         | 1.78   | 1.78    | 1.78   | 1.78    | 1.78   | 1.78    | 1.78   | 1.74    | 1.72   | 1.69    | 1.28   |
| wetted cross section:             | 0.349   | 0.349  | 0.349   | 0.349  | 0.349   | 0.349  | 0.349   | 0.331  | 0.317   | 0.301  | 0.11    |        |
| Hydraulic radius:                 |         | 0.197  | 0.197   | 0.197  | 0.197   | 0.197  | 0.197   | 0.197  | 0.19    | 0.185  | 0.179   | 0.0857 |

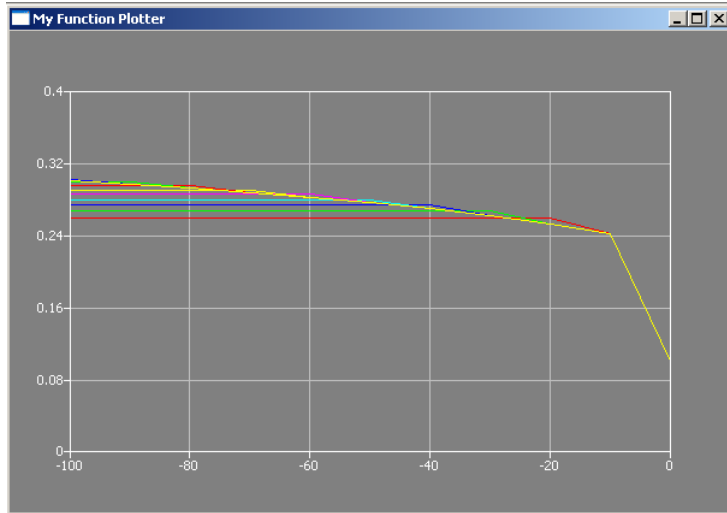
$$h_{k+1} = h_k - \frac{f(h_k)}{f'(h_k)} = h_k - \frac{N}{D} \quad (16)$$

```
1 // Newton iteration loop
2 for(int k=0;k<kn;k++)
3 {...
4     RunNewtonStep();
5     ...}
```

$$\text{bis } \| h_{k+1} - h_k \| < \epsilon \quad (17)$$

# Anlage BHYWI-08-11C Gerinnehydraulik GUI





# Anlage

## EX10-gerinne-python

### Gerinnehydraulik

### Newton-Verfahren

Hydroinformatics II (Olaf Kolditz)

Exercise BHYWI-08-12-for-python: Newton-Verfahren Gerinnehydraulik

