# Hydroinformatik - SoSe 2025

# UW-BHW-414-H1: Grundlagen der Kontinuumsmechanik

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Dresden, 06.06.2025

TECHNISCHE UNIVERSITÄT DRESDEN

# Zeitplan: Hydroinformatik I+II

Sommersemester 2025

Stand: 08.05.2025

| Nr. | KW | Datum | ID | Thema |
|---|---|---|---|---|
| 01 | 15 | 11.04.2025 | UW-BHW-414-A | Einführung, Werkzeuge#1, Hello World |
| 03 | 17 | 25.04.2025 | UW-BHW-414-B | Umweltinformatik, Werkzeuge#2 (git), Datentypen |
| 05 | 18 | 02.05.2025 | UW-BHW-414-C | Selbststudium |
| 07 | 19 | 09.05.2025 | UW-BHW-414-D | Objekt-Orientierte Programmierung: C++, Klassen |
| 09 | 20 | 16.05.2025 | UW-BHW-414-E | Python |
| 11 | 21 | 23.05.2025 | UW-BHW-414-F | Modellierung, Digitalisierung, Wasser 4.0 |
| 13 | 22 | 30.05.2025 | UW-BHW-414-G | KI, Maschinelles Lernen, Neuronale Netzwerke |
| 15 | 23 | 06.06.2025 | UW-BHW-414-H | Kontinuumsmechanik, Hydromechanik |
| 05 | 18 | 13.06.2025 | | Vorlesungsfreie Woche |
| 17 | 25 | 20.06.2025 | UW-BHW-414-I | Differentialgleichungen, Näherungsverfahren |
| 19 | 26 | 27.06.2025 | UW-BHW-414-J | Finite-Differenzen, explizite Verfahren |
| 21 | 27 | 04.07.2025 | UW-BHW-414-K | Finite-Differenzen, implizite Verfahren |
| 24 | 28 | 11.07.2025 | UW-BHW-414-L | Gerinnehydraulik, Grundwasserhydraulik |
| 25 | 29 | 18.07.2025 | UW-BHW-414-M | Zusammenfassung, Klausurvorbereitung |

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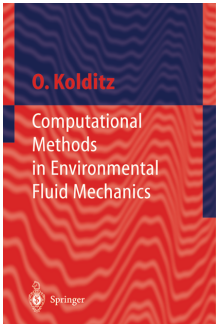
# Fahrplan für heute ...

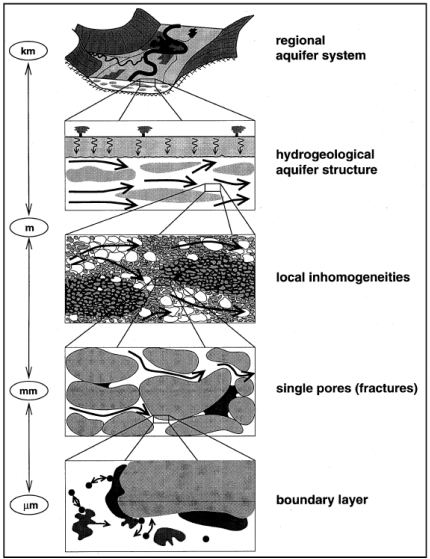
- Motivation
- Lagrange Konzept
- Euler Konzept
- Reynolds Transport Theorem
- Fluxes
- Bilanzgleichungen
- Erhaltungsgrößen

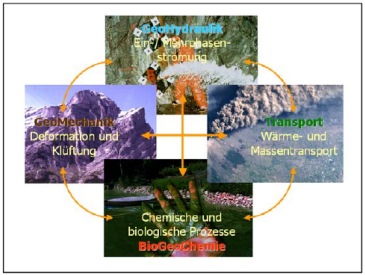


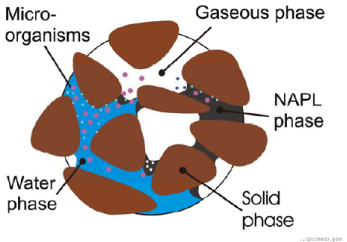

>> Skript

$\Longrightarrow$ viel Theorie - vor allem die mathematische Schreibweise verstehen ”zu lesen”

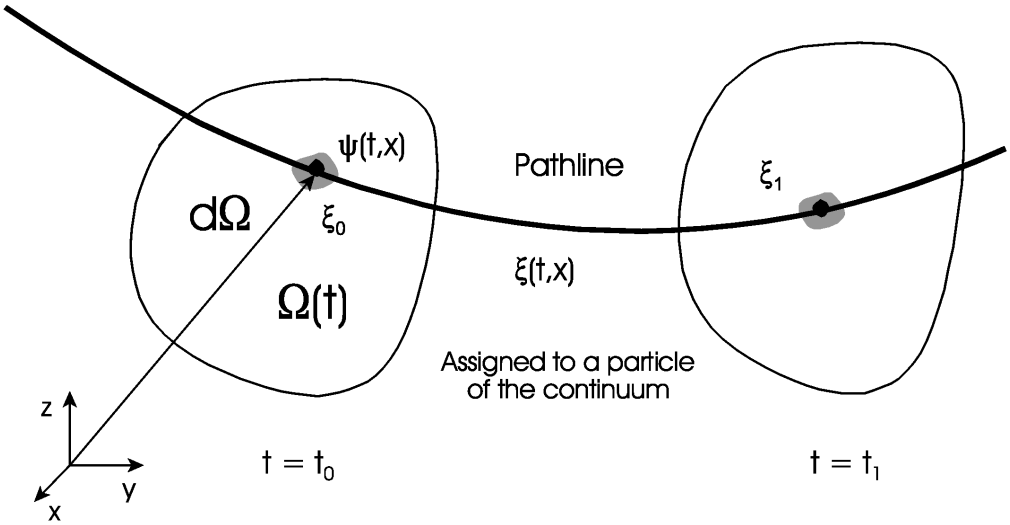
# Skalen


km
m
mm
µm
regional
aquifer system
hydrogeological
aquifer structure
local inhomogeneities
single pores (fractures)
boundary layer

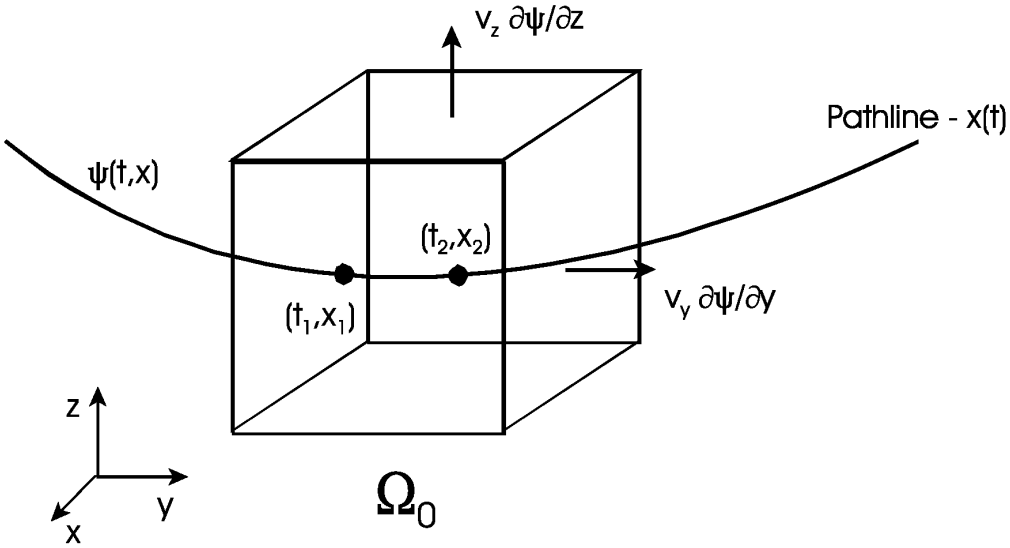

Geohydraulik
Ein-/ Mehrphasen-
strömung
GeoMechanik
Deformation und
Klüftung
Transport
Wärme- und
Massentransport
Chemische und
biologische Prozesse
BioGeoChemie


Micro-
organisms
Gaseous phase
NAPL
phase
Water
phase
Solid
phase

# Lagrange Konzept (1.1.1)



# Euler Konzept (1.1.1)



- Volumenintegral (Zeichnung)

$$\int_{\Omega} d\Omega \tag{1}$$

$$\int_{a}^{b} f(x)dx = \lim_{(x_{k+1}-x_k)\to 0} \sum_{k=1}^{\infty} \left(f(x_{k+1} - f(x_k)\right)\left(x_{k+1} - x_k\right) \tag{2}$$

- Oberflächen-(Ring)-Integral (Zeichnung)

$$\oint_{\partial\Omega} \mathbf{n} \cdot d\mathbf{S} \tag{3}$$

- Materielle Ableitung

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \mathbf{v} \cdot \nabla\psi \tag{4}$$

- Gradient (Vektor)

$$\nabla = \{\partial/\partial x, \partial/\partial y, \partial/\partial z\} \tag{5}$$

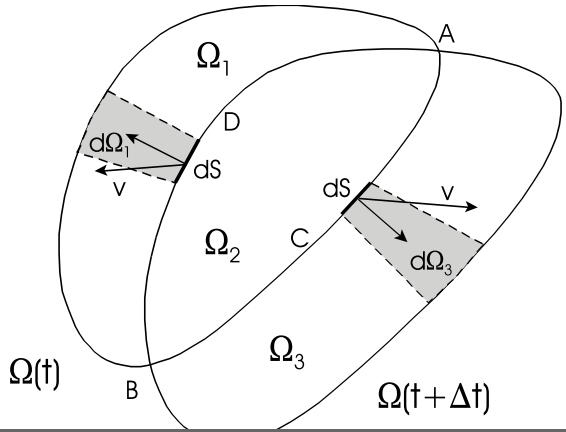
- Divergenz (Skalar)

$$\nabla \cdot \mathbf{v} = \partial v_x/\partial x + \partial v_y/\partial y + \partial v_z/\partial z \tag{6}$$

Aufgabe HA01: Was ist $\mathbf{v} \cdot \nabla\psi$?

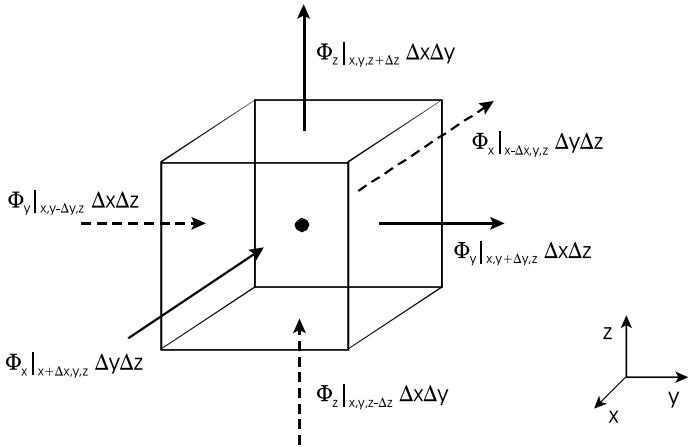
# Reynolds Transport Theorem (Lagrange) (1.1.3)

$$\frac{d}{dt}\int_{\Omega}\psi d\Omega = \int_{\Omega}\frac{\partial\psi}{\partial t}d\Omega + \oint_{\partial\Omega}\psi(t)\mathbf{v}\cdot d\mathbf{S} = \int_{\Omega} q^{\psi} d\Omega \qquad (7)$$




Beweisführung: Siehe Skript Abschn. 1.1.3

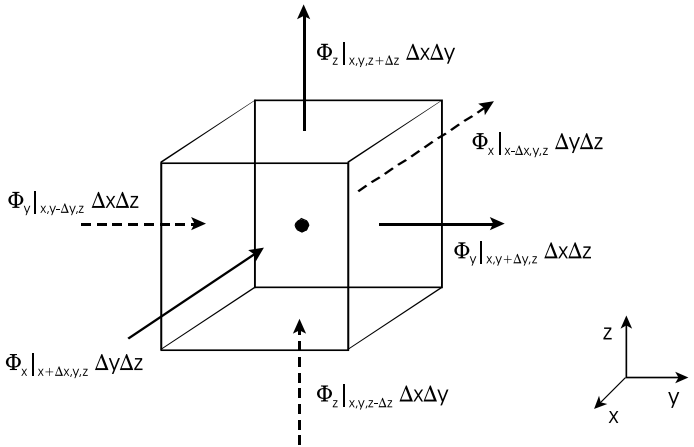
# Reynolds Transport Theorem (Euler) (1.1.3)




$$\Phi_x^{\psi} = \partial\psi/\partial x \quad , \quad \Phi^{\psi} = \nabla\psi$$

Frage: Ist $\Phi^{\psi}$ eine skalare oder vektorielle Größe ?

# Reynolds Transport Theorem (Euler) (1.1.3)




$$\oint_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega \qquad (8)$$

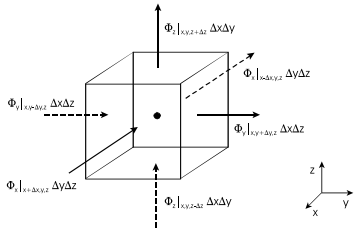
$$\oint_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega \tag{9}$$

$$\lim_{\Omega \to 0} \frac{1}{\Omega} \oint_{\partial\Omega} \Phi \cdot d\mathbf{S} = \nabla \cdot \Phi \tag{10}$$

**Point-Flux**

# Reynolds Transport Theorem (Euler) (1.1.3)

## General Balance Equation




$$\frac{d}{dt}\int_{\Omega}\psi d\Omega = \underbrace{\frac{\partial}{\partial t}\int_{\Omega}\psi d\Omega}_{1} + \underbrace{\oint_{\partial\Omega}\Phi^{\psi}\cdot d\mathbf{S}}_{2} = \underbrace{\int_{\Omega}q^{\psi}d\Omega}_{3} \tag{11}$$

with:

1. Rate of change of total amount of quantity $\psi$ in the control volume,
2. Net rate of increase / decrease of $\psi$ due to fluxes,
3. Rate of increase / decrease of $\psi$ due to sources.

# Reynolds Transport Theorem (Euler) (1.1.3)

$$\frac{d}{dt}\int_{\Omega}\psi d\Omega = \underbrace{\frac{\partial}{\partial t}\int_{\Omega}\psi d\Omega}_{1} + \underbrace{\oint_{\partial\Omega}\Phi^{\psi}\cdot d\mathbf{S}}_{2} = \underbrace{\int_{\Omega} q^{\psi} d\Omega}_{3} \tag{12}$$

using

$$\oint_{\partial\Omega}\Phi^{\psi}\cdot d\mathbf{S} = \int_{\Omega}\nabla\cdot\Phi^{\psi} d\Omega \tag{13}$$

$$\frac{d}{dt}\int_{\Omega}\psi d\Omega = \int_{\Omega}\frac{\partial\psi}{\partial t} d\Omega + \int_{\Omega}\nabla\cdot\Phi^{\psi} d\Omega = \int_{\Omega} q^{\psi} d\Omega \tag{14}$$

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# Reynolds Transport Theorem

$$\frac{d}{dt}\int_{\Omega}\psi d\Omega = \int_{\Omega}\frac{\partial\psi}{\partial t}d\Omega + \int_{\Omega}\nabla\cdot\Phi^{\psi}d\Omega = \int_{\Omega}q^{\psi}d\Omega \tag{15}$$

$$\forall\Omega : \frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla\cdot\Phi^{\psi} = q^{\psi} \tag{16}$$

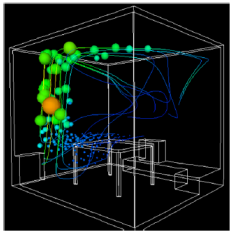
# Reynolds Transport Theorem

$$\frac{d}{dt}\int_{\Omega}\psi d\Omega = \int_{\Omega}\left(\frac{\partial\psi}{\partial t} + \nabla\cdot\mathbf{\Phi}^{\psi}\right)d\Omega = \int_{\Omega} q^{\psi} d\Omega$$

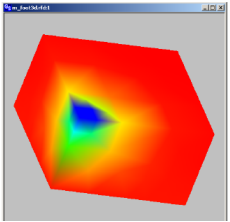
$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \mathbf{v}^{E}\nabla\psi$$

Lagrange

Euler



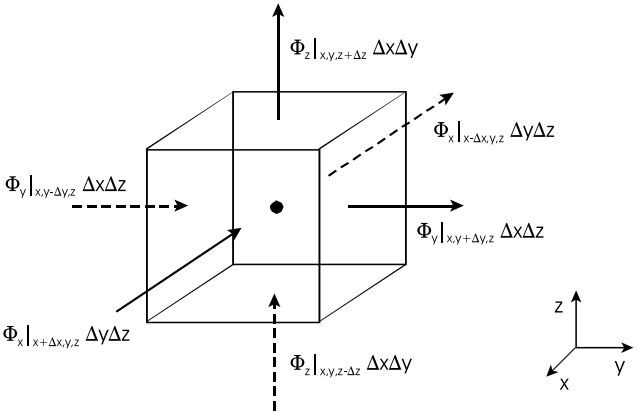
http://www.cscs.ch/~mvalle/Libro/



# Fluxes (1.1.6)

The total flux $\Phi^{\psi}$ of a quantity $\psi$ is defined as

$$\Phi^{\psi} = \mathbf{v}^{E}\psi \tag{17}$$



# Fluxes (1.1.6)

$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \nabla \cdot \Phi^{\psi} = q^{\psi} \tag{18}$$

$$\Phi^{\psi} = \mathbf{v}^{E} \psi \tag{19}$$

$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \nabla \cdot (\mathbf{v}^{E} \psi) = q^{\psi} \tag{20}$$

# Fluxes (1.1.6)

$$\Phi^{\psi} = \mathbf{v}^{E}\psi = \underbrace{\mathbf{v}\psi}_{\Phi_A^{\psi}} + \underbrace{(\mathbf{v}^{E} - \mathbf{v})\psi}_{\Phi_D^{\psi}} \tag{21}$$

and, therefore, decomposed into two parts: an advective flux $\Phi_A^{\psi}$ and a diffusive flux $\Phi_D^{\psi}$ relative to the mass-weighted velocity:

- advective flux of quantity $\psi$

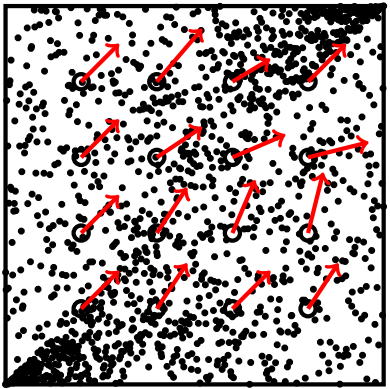
$$\Phi_A^{\psi} = \mathbf{v}\psi \tag{22}$$

- diffusive flux of quantity $\psi$ (Fick's law)

$$\Phi_D^{\psi} = -\mathbf{D}^{\psi}\nabla\psi \tag{23}$$

# Fluxes (1.1.6)

## Velocities



$$\mathbf{v}^E = \cup \mathbf{v}_i$$

$$\mathbf{v}^E = \mathbf{v} + \hat{\mathbf{v}}$$

$$\hat{\mathbf{v}} = \mathbf{v}^E - \mathbf{v}$$

# General Balance Equation (1.1.7)

- Integral form

$$\int_\Omega \frac{d\psi}{dt} = \int_\Omega \frac{\partial \psi}{\partial t} + \int_\Omega \nabla \cdot (\mathbf{v}\psi) - \int_\Omega \nabla \cdot (\mathbf{D}^\psi \nabla \psi) = \int_\Omega Q^\psi \tag{24}$$

- Differential form

$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \nabla \cdot (\mathbf{v}\psi) - \nabla \cdot (\mathbf{D}^\psi \nabla \psi) = Q^\psi \tag{25}$$

# Conservation Quantities (1.1.2)

The amount of a quantity in a defined volume $\Omega$ is given by

$$\Psi = \int_{\Omega} \psi d\Omega(t) \tag{26}$$

where $\Psi$ is an extensive conservation quantity (i.e. mass, momentum, energy) and $\psi$ is the corresponding intensive conservation quantity such as mass density $\rho$, momentum density $\rho\mathbf{v}$ or energy density $e$.

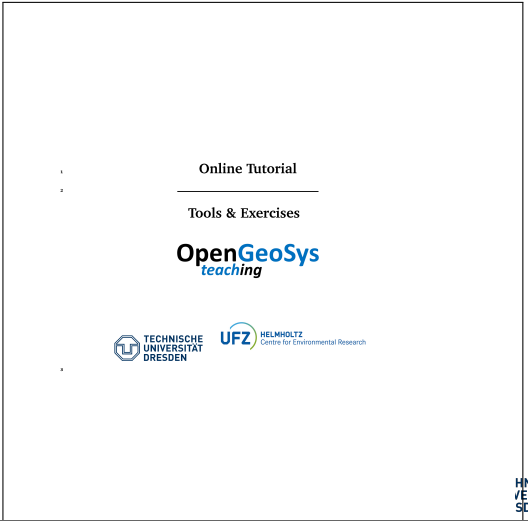
| Extensive quantity | Symbol | Intensive quantity | Symbol |
|---|---|---|---|
| Mass | $M$ | Mass density | $\rho$ |
| Linear momentum | $\mathbf{m}$ | Linear momentum density | $\rho\mathbf{v}$ |
| Energy | $E$ | Energy density | $e = \rho i + \frac{1}{2}\rho v^2$ |

# Übung
# Git: Übungen klonen

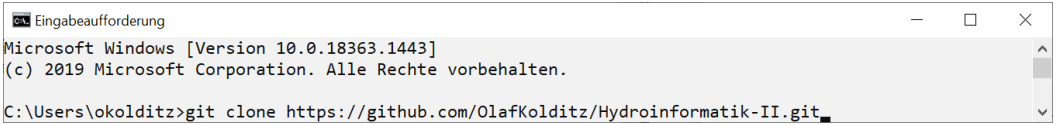
- EX01: Compiler installieren ✓
- EX02: Python installieren (Pfadvariable!) ✓
- EX03: Git installieren ✓
- EX04: Git: Übungen klonen

# OGS-Teaching Tutorial

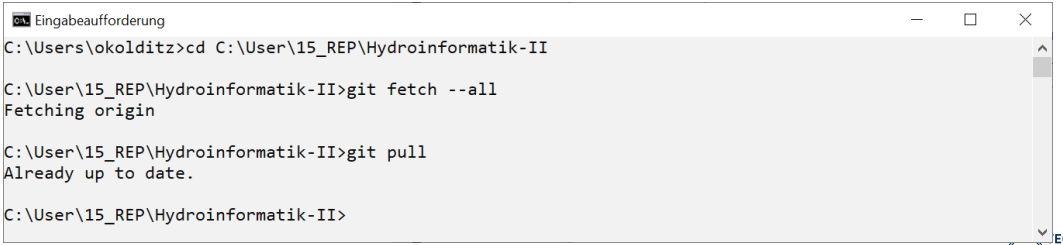
## Git: Übungen klonen



## 1.1.3 Cloning sources from a `git` repository




## 1.1.4 Updating sources from a `git` repository



# Hydroinformatik - SoSe 2025

# UW-BHW-414-H2: Grundlagen der Hydromechanik

Prof. Dr.-Ing. habil. Olaf Kolditz

[1]Helmholtz Centre for Environmental Research – UFZ, Leipzig

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Sommersemester 2025

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| 24 | 28 | 11.07.2025 | UW-BHW-414-L | Gerinnehydraulik, Grundwasserhydraulik |
| 25 | 29 | 18.07.2025 | UW-BHW-414-M | Zusammenfassung, Klausurvorbereitung |

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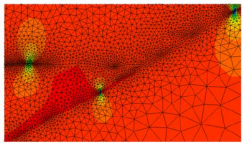
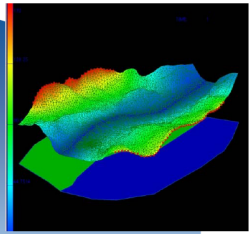
# Konzept

$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \mathbf{v}^E \nabla \psi$$

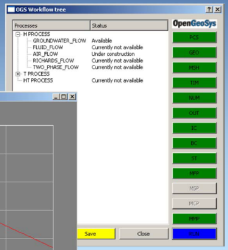


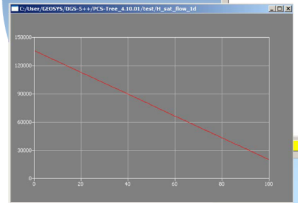
Basics
Mechanik

Anwendung



Numerische
Methoden



Programmierung
Visual C++

Prozessverständnis

# Fahrplan für heute

- Erhaltungsgrößen
- Massenerhaltung
- Fluidmassenerhaltung
- Diffusion
- Impulserhaltung
- Spannungen
- Fluiddruck
- Strömungsprobleme

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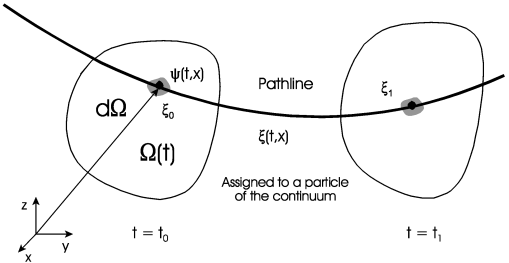
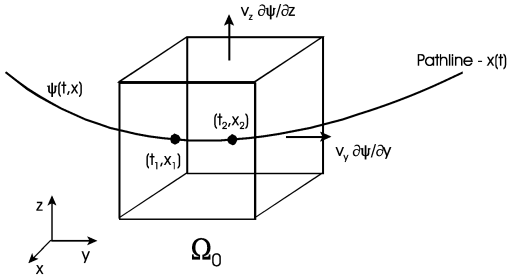
## Lagrange




## Euler



$$\begin{aligned} \frac{d}{dt}\int_\Omega \psi \, d\Omega &= \frac{\partial}{\partial t}\int_\Omega \psi \, d\Omega + \oint_{\partial\Omega} \mathbf{\Phi}^\psi \cdot d\mathbf{S} \\ &= \frac{\partial}{\partial t}\int_\Omega \psi \, d\Omega + \int_\Omega \nabla \cdot \mathbf{\Phi}^\psi d\Omega \end{aligned} \tag{1}$$

$\lim d\Omega \rightarrow 0$

$$\begin{aligned} \frac{d\psi}{dt} &= \frac{\partial \psi}{\partial t} + \nabla \cdot \mathbf{\Phi}^\psi \\ &= \frac{\partial \psi}{\partial t} + \nabla \cdot (\mathbf{v}\psi) - \nabla \cdot (\mathbf{D}^\psi \nabla \psi) \\ &= Q^\psi \end{aligned} \tag{2}$$

HA 02#2020: Komponentenschreibweise $\nabla \cdot (\mathbf{v}\psi)$, $\nabla \cdot (\mathbf{D}^\psi \nabla \psi)$

The amount of a quantity in a defined volume $\Omega$ is given by

$$\Psi = \int_{\Omega} \psi d\Omega(t) \tag{3}$$

where $\Psi$ is an extensive conservation quantity (i.e. mass, momentum, energy) and $\psi$ is the corresponding intensive conservation quantity such as mass density $\rho$, momentum density $\rho\mathbf{v}$ or energy density $e$.

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| Linear momentum | $\mathbf{m}$ | Linear momentum density | $\rho\mathbf{v}$ |
| Energy | $E$ | Energy density | $e = \rho i + \frac{1}{2}\rho v^2$ |

# (Phase) Mass Conservation

$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \nabla \cdot (\mathbf{v}\psi) - \nabla \cdot (\mathbf{D}^{\psi} \nabla \psi) = Q^{\psi} \tag{4}$$

The differential equation of mass conservation in divergence form becomes

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \nabla \cdot (\mathbf{v}\rho) = 0 \tag{5}$$

Partial differentiation of the above equation gives

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{v} = 0 \tag{6}$$

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# (Phase) Mass Conservation

Using the material (or convective) derivative the mass conservation equation can be rewritten as

$$\frac{\partial \rho}{\partial t} = -\rho \nabla \cdot \mathbf{v} \tag{7}$$

Note, above convective form of mass conservation equation becomes zero only for incompressible flows, i.e.

$$\frac{\partial \rho}{\partial t} = 0 \tag{8}$$

requires divergence-free flow.

$$\nabla \cdot \mathbf{v} = 0 \tag{9}$$

From eqn. (6) results that the above expression is the continuity equation for a homogeneous fluid ($\rho = \text{const}$).

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# Divergenzfreie Strömung

$$\nabla \cdot \mathbf{v} = 0 \tag{10}$$

Links:

- https://de.wikipedia.org/wiki/Stromfunktion
- https://www.ingenieurkurse.de/stroemungslehre/ebene-stroemungen/quelle-und-senke-divergenz.html

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# Divergenzfreie Strömung

## Übung: HyBHW-1-02-04-E1

```python
import numpy as np
import matplotlib.pyplot as plt
# set up a normalized grid:
dim= 20
xarray= np.arange(-dim,dim)
yarray= np.arange(-dim,dim)
# (fluid) flow from a source at L to a sink at -L:
L = dim/2
x,y = np.meshgrid(xarray,yarray)
vx = (x-L)/((x-L)**2+y**2) - (x+L)/((x+L)**2 +y**2)
vy = y/((x-L)**2+y**2) - y/((x+L)**2 +y**2)
# plot the flow lines:
plt.figure()
plt.quiver(x,y, vx, vy, pivot='mid')
plt.xlabel("$x$-axis")
plt.ylabel("$y$-axis")
plt.axis('equal')
plt.show()
```

**Listing:** Python code for divergence-free flow - div v

Source: University of Auckland

# Übungen: Hydroinformatik

Hydroinformatik II - HyBHW-1-02 https://github.com/OlafKolditz/HYDROINFORMATIK-II

- EX01: Jupyter Notebook
- EX02: Python: matplotlib
- EX03: Kontinnumsmechanik: Skalarprodukt
- EX04: Hydromechanik: Divergenzfreie Strömung
- EX05: Analytische Lösung: Elliptische Gleichung
- EX06: Analytische Lösung: Parabolische Gleichung (Diffusion)
- EX07: Analytische Lösung: Transportgleichung (ADE)
- EX08: Finite-Differenzen-Methode (FDM) explizit
- EX09: Finite-Differenzen-Methode (FDM) implizit
- EX10: Gerinnehydraulik

```
Quellcode
...
```

**Listing:** Quellcode für Übungen (C++ und Python)

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# Conservation Quantities (1.1.2)

The amount of a quantity in a defined volume $\Omega$ is given by

$$\Psi = \int_{\Omega} \psi d\Omega(t) \tag{11}$$

where $\Psi$ is an extensive conservation quantity (i.e. mass, momentum, energy) and $\psi$ is the corresponding intensive conservation quantity such as mass density $\rho$, momentum density $\rho\mathbf{v}$ or energy density $e$.

| Extensive quantity | Symbol | Intensive quantity | Symbol |
|---|---|---|---|
| Mass | $M$ | Mass density | $\rho$ |
| Linear momentum | $\mathbf{m}$ | Linear momentum density | $\rho\mathbf{v}$ |
| Energy | $E$ | Energy density | $e = \rho i + \frac{1}{2}\rho v^2$ |

# Momentum Conservation

$\psi = \rho \mathbf{v}$

$$\frac{\partial}{\partial t} \int_{\Omega} \rho \mathbf{v} d\Omega + \oint_{\partial\Omega} \mathbf{\Phi}^m \cdot d\mathbf{S} = \int_{\Omega} \rho \mathbf{f} d\Omega \tag{12}$$

Flux term: The advective momentum flux is defined as

$$\mathbf{\Phi}^m = (\rho \mathbf{v}) \otimes \mathbf{v} = (\rho \mathbf{v}) \mathbf{v} \tag{13}$$

$$\mathbf{F} = \int_{\Omega} \rho \mathbf{f} d\Omega = \int_{\Omega} \rho (\mathbf{f}^e + \mathbf{f}^i) d\Omega = \underbrace{\int_{\Omega} \rho \mathbf{f}^e d\Omega}_{\text{External forces}} + \underbrace{\oint_{\partial\Omega} \boldsymbol{\sigma} : d\mathbf{S}}_{\text{Internal forces}} \tag{14}$$

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# Momentum Conservation

Substituting now flux and source terms of momentum we obtain

$$\frac{\partial}{\partial t}\int_{\Omega}\rho\mathbf{v}d\Omega+\oint_{\partial\Omega}\rho\mathbf{v}(\mathbf{v}\cdot d\mathbf{S})=\int_{\Omega}\rho\mathbf{f}^{e}d\Omega+\oint_{\partial\Omega}\boldsymbol{\sigma}d\mathbf{S} \tag{15}$$

Applying the Gauss-Ostrogradskian theorem to the surface integrals

$$\begin{aligned}\oint_{\partial\Omega}\rho\mathbf{v}(\mathbf{v}\cdot d\mathbf{S}) &= \int_{\Omega}\nabla\cdot(\rho\mathbf{v}\mathbf{v})\,d\Omega\\ \oint_{\partial\Omega}\boldsymbol{\sigma}:d\mathbf{S} &= \int_{\Omega}\nabla\cdot\boldsymbol{\sigma}\,d\Omega\end{aligned} \tag{16}$$

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# Momentum Conservation

The differential form of the momentum conservation law is then

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v}\mathbf{v}) = \rho \mathbf{f}^e + \nabla \cdot \boldsymbol{\sigma} \tag{17}$$

The above equation is now extended by partial integration

$$\begin{aligned} &\rho \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\frac{\partial \rho}{\partial t} + (\rho \mathbf{v}) \cdot \nabla \mathbf{v} + \mathbf{v}\nabla \cdot (\rho \mathbf{v}) \\ &= \rho[\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}] + \mathbf{v}[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v})] \\ &= \rho \mathbf{f}^e + \nabla \cdot \boldsymbol{\sigma} \end{aligned} \tag{18}$$

Using the mass conservation equation (5) and dividing by $\rho$ we obtain

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{f}^e + \frac{1}{\rho}\nabla \cdot \boldsymbol{\sigma} \tag{19}$$

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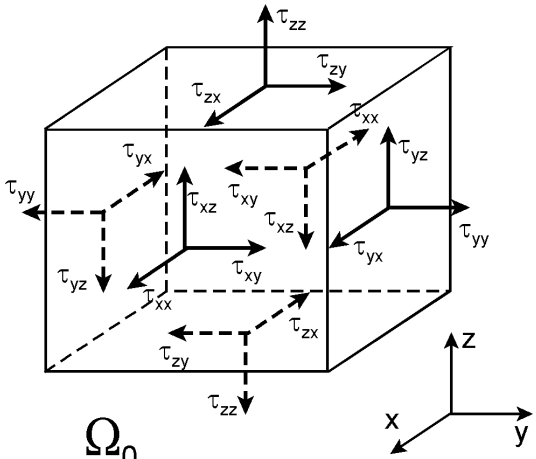
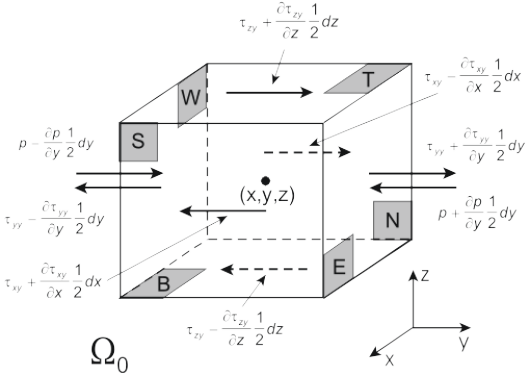
# Momentum Conservation: Stress Tensor

$$\boldsymbol{\sigma} = -p\mathbf{I} + \boldsymbol{\tau} \quad , \quad \boldsymbol{\tau} = \nu \nabla \mathbf{v} \tag{20}$$



# Momentum Conservation: Stress Tensor

$$\boldsymbol{\tau} = \nu \nabla \mathbf{v} \tag{21}$$



# Fluid Momentum Balance

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{f}^e + \frac{1}{\rho}\nabla \cdot \boldsymbol{\sigma} \tag{22}$$

In index notation the above vector equation is written as

$$\begin{aligned}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} &= \frac{1}{\rho}\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}\right) \\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} &= \frac{1}{\rho}\left(\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z}\right) \\
\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} &= g + \frac{1}{\rho}\left(\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\right)
\end{aligned} \tag{23}$$

with $u = v_x, v = v_y, w = v_z$ and $\mathbf{f}^e = \mathbf{g}$.

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# Flow Equations - Systematic

Stress Tensor

$$\boldsymbol{\sigma} = -p\mathbf{I} + \boldsymbol{\tau} \tag{24}$$

Navier-Stokes Equation

$$\boxed{\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{f}^e - \frac{1}{\rho}\nabla p + \nu \Delta \mathbf{v}} \tag{25}$$

Euler Equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{f}^e - \frac{1}{\rho}\nabla p \tag{26}$$

Stokes Equation

$$\frac{\partial \mathbf{v}}{\partial t} = \mathbf{f}^e - \frac{1}{\rho}\nabla p + \nu \Delta \mathbf{v} \tag{27}$$

Darcy Equations

$$0 = \mathbf{f}^e - \frac{1}{\rho}\nabla p + \nu \Delta \mathbf{v}$$

# (Component) Mass Conservation

$$\psi = \rho_k = C_k \tag{29}$$

$$\frac{dC_k}{dt} = \frac{\partial C_k}{\partial t} + \nabla \cdot (\mathbf{v} C_k) - \nabla \cdot (\mathbf{D_k} \nabla C_k) = Q_k \tag{30}$$

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# Übungen und Hausaufaben

- EX: Übungen
- HA: Hausaufgaben

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# Übungen: Hydroinformatik

Hydroinformatik II - HyBHW-1-02 https://github.com/OlafKolditz/HYDROINFORMATIK-II

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```
Quellcode
...
```

**Listing:** Quellcode für Übungen (C++ und Python)

TECHNISCHE UNIVERSITÄT DRESDEN

# Hausaufgaben

# Hausaufgaben: Hydroinformatik

## Hydroinformatik II - HyBHW-1-02

1. Skalarprodukt: Schreiben sie das Skalarprodukt $\nabla \cdot \mathbf{v}$ in Komponentenschreibweise.
2. Mechanik: Was ist $\mathbf{v} \cdot \nabla\psi$? Physikalische Bedeutung des Terms
3. Mechanik: Was ist $\Phi^{\psi}$? Physikalische Bedeutung des Terms
4. Hydromechanik: Komponentenschreibweise $\nabla \cdot (\mathbf{v}\psi)$
5. Hydromechanik: Komponentenschreibweise $\nabla \cdot (\mathbf{D}^{\psi}\nabla\psi)$
6. Analytik: Prüfen Sie die Gültigkeit einer der Lösungen für die Diffusionsgleichung: (20), (21), (22) (siehe Vorlesung 5 >> HyBHW-S2-01-V05).
7. Analytik: Stellen Sie die ausgewählte analytische Lösung für die 1-D parabolische Differentialgleichung unter Verwend der Übung EX06-parabolische-gleichung-1D.py dar. Ergänzen Sie Ihren Namen oder Matrikelnummer mit dem Befehl `plt.title("Name oder Matrikelnummer")`.
8. Numerik: Darstellung der numerischen Lösung (explizite FDM) für die 1-D parabolische Differentialgleichung (EX08-fdm-explicit-python). Produzieren Sie eine stabile und instabile Lösung.
9. Numerik: Darstellung der numerischen Lösung (implizite FDM) für die 1-D parabolische Differentialgleichung (EX09-fdm-implicit-python). Produzieren Sie die stationäre Lösung.

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# Hausaufgaben: Hydroinformatik

## Beispiel: Aufgaben 7-9

- zum Internet-Repository gehen (Webseite)
- Python-File editieren (Matrikel-Nummer oder Name)
- Programme zum Rechnen und Darstellen ausführen
- Ergebnis (Abbildung) in die Hausaufgaben einfügen

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